Problem 339. Posed by G.O.H. Katona, Budapest
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Let P be a graded poset of rank R in which P_i is the set of elements with rank i and $N_i = |P_i|$. Given an antichain A in P, let $f_i = |A \cap P_i|$. The poset P has the *LYM property* if $\sum_i f_i/N_i \leqslant 1$ for every antichain A. The poset P has the *normalized matching property* if, for all i, every t elements in P_i together cover at least $(N_{i-1}/N_i)t$ elements in P_{i-1}.

Kleitman [1] proved that the LYM property and normalized matching property are equivalent.

The *profile* of an antichain A is the vector $(f_0, \ldots, f_R)$. For a graded poset P, an *extreme antichain profile* (EAP) is an extreme point of the convex hull of the set of profiles of antichains in P.

It is easy to show that the extreme points of the convex hull of profiles of antichains contained in two consecutive ranks have at most one nonzero component if and only if the normalized matching property holds. Thus, the theorem of Kleitman can be reformulated as follows: P has the LYM property if and only if every EAP of P has at most one nonzero coordinate. This behavior does not extend for analogous restrictions on larger sets of coordinates.

Problem. Suppose that P has no EAP that is nonzero in exactly three coordinates. What is the maximum possible number of nonzero coordinates in an EAP of P?

Sali gave an example showing that the answer exceeds 2.

References

[1] D.J. Kleitman, On an extremal property of antichains in partial orders: The *LYM* property and some of its implications and applications. Combinatorics (Proceedings of the NATO Advanced Study Institute, Breukelen, 1974), Part 2: Graph Theory; Foundations, Partitions and Combinatorial geometry, Math. Centre Tracts, No. 56, Math. Centrum, Amsterdam, 1974, pp. 77–90.