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RÉNYI AND THE COMBINATORIAL SEARCH PROBLEMS

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1. Introduction

The whole story has started with the Hillman, Rényi's car. It was a kind of a member of his family. That time, in the early sixties, very few people had cars in Hungary, the gasoline was cheap, there was no parking problem. Once he gave me a ride from the Institute to the cafeteria, less than 200 meters. One day, however, the Hillman stopped its smooth services. Obviously, there was some electrical problem with it. The electrician, however, was unable to find the source of the trouble. Rényi had to find it, himself. He has found it, and in the mean-time he has developed a general mathematical model for the situation.

The car can be considered as a finite set of its parts. The car does not work since one (hopefully exactly one, so we suppose it) of its parts does not work properly. When trying to find it, tests are performed. One test tries to function a subset of the parts. If it does not work then the defective part is in this subset, otherwise it is not contained in it. After performing several such tests we have to determine the defective part.

Let us formulate it a little more mathematically. A finite set X of n elements is given. A distinguished element x of X is given but it is unknown by us. Furthermore, a family $\mathcal{A}$ of subsets of X is given. We can ask the questions if "x is in A" or not, for members A of $\mathcal{A}$. We have to identify x on the basis of the answers for the above questions. We call the members of $\mathcal{A}$ *question sets*. They are the potential questions.

There are two basically different models. If the sequence $A_1, \ldots, A_m$ of actual questions (a subfamily of $\mathcal{A}$) is fixed in advance then we say that this is a *linear search*. The obvious mathematical aim is to minimize the number m of questions. On the other hand, the choice of the next question may depend on the answers to the previous questions. The first question set is

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$A \in \mathcal{A}$. If the answer is $x \notin A$ then the second question set is $A_0 \in \mathcal{A}$ while in the case of the answer $x \in A$ the second question set is $A_1 \in \mathcal{A}$. The third question set is $A_{00} \in \mathcal{A}$, $A_{01} \in \mathcal{A}$, $A_{10} \in \mathcal{A}$ and $A_{11} \in \mathcal{A}$, resp. depending on the two previous answers, etc. This is called a *tree search*. In this case the maximum number of questions (the length of the longest path of the tree) should be minimized. The tree search seems to be more practical, however the linear search is much simpler to organize, so in the era of fast computers it might be equally important.

With this model Rényi [27] initiated an area, the combinatorial search problems. He and his followers have written many papers. However his work was not the only source of these investigations. Now we show some other sources.

Let X be a set of soldiers in World War II. A sample of blood is drawn from every person. The ones containing *syphilitic antigen* should be found using the *Wasserman test*. It was an original idea that the blood samples could be poured and tested together. In this way it can be decided if a certain subset of soldiers contains an infected person or not. This model is basicly identical with the previous one, the only difference is that the number of elements to be identified is not known, in advance. (See [10] and [31].) This kind of models are called group testing and considered to belong to Mathematical Statistics.

An even older question was raised by Steinhaus [30]. A set of n table tennis players is given. Suppose that their abilities are constant, it can be described with a real number, there is no randomness, so the better one always defeats the weaker one. The aim is to determine their total order by pairwise comparisons, that is, table tennis matches. Although it is not clear at the first sight, this problem is also covered by the above model. Let X be the set of $n!$ permutations of the players. One of these permutations is searched. One match determines if this unknown permutation belongs to the set of $n!/2$ permutations where player a is better than player b.

So, one can say that the area has three sources (Fig. 1). The present author wrote a survey paper [16] containing 66 references, the book by Ahlswede and Wegener [1] has 166 references and finally the most recent summarization of the area, the book of Aigner [2] quotes 198 papers. These numbers show that the area became quite large, a small paper cannot survey it. Therefore the aim of the present paper is to survey those papers which were written (mostly by Hungarian authors) under a direct or an indirect influence of Rényi.

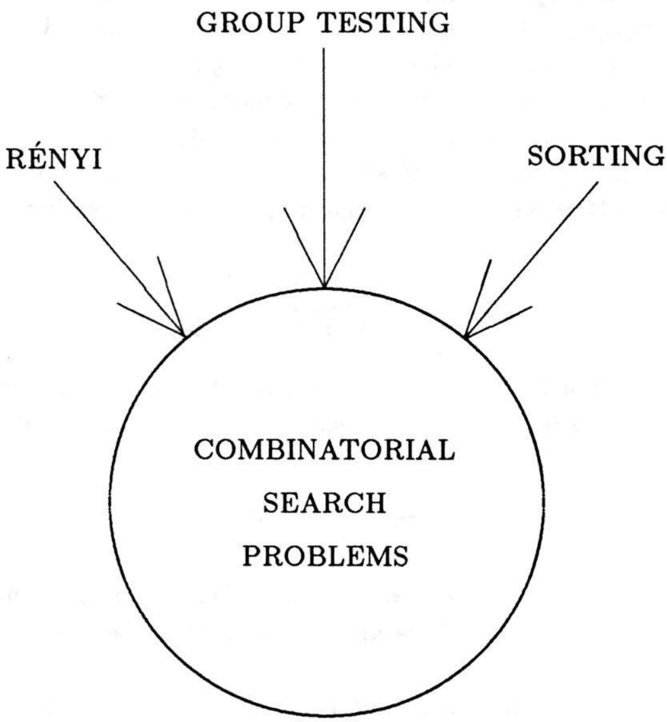

Fig. 1

2. Qualitatively independent sets and partitions

Let $A, B \subset X$ be two question sets. Suppose that they are disjoint. First ask if the unknown x is in A or not. If the answer is no, we have to ask B, as well. However, if the answer is yes then there is no need to ask B, we know that x is not in B. If one of

$$A \cup B, \quad A \cup \overline{B}, \quad \overline{A} \cup B, \quad \overline{A} \cup \overline{B} \tag{1}$$

is empty, the situation is similar, that is, one of the possible answers to the first question makes the second question superfluous. We say that A and B are *qualitatively independent* if none of the sets in (1) is empty.

Rényi [28] raised the question what is the maximum of $|\mathcal{A}|$ on an n-element set if any two different members of $\mathcal{A}$ are qualitatively independent.

The answer is very easy for even n-s. It is easy to see that the sets in (1) are all non-empty iff there is no inclusion among the sets $A, \overline{A}, B, \overline{B}$.
A family S of subsets is called a *Sperner family* iff there is no inclusion in it, that is, $C \not\subseteq D$ holds for any two distinct members of S. Using this notion, we can state that any two members of $\mathcal{A} = \{A_1, A_2, \dots, A_m\}$ are qualitatively independent if and only if $\mathcal{A}^* = \{A_1, A_2, \dots, A_m, \overline{A}_1, \overline{A}_2, \dots, \overline{A}_m\}$ is a Sperner family. However, the maximum size of a Sperner family is known:

$$\binom{n}{\lfloor \frac{n}{2} \rfloor}.$$

Therefore, if $\mathcal{A}$ is a family of pairwise qualitatively independent sets then $\mathcal{A}^*$ is a Sperner family and $2m$ is less than equal to the above binomial coefficient, so

$$m \le \frac{1}{2}\binom{n}{\lfloor \frac{n}{2} \rfloor}.$$

This inequality is true for any n but it is also sharp for even n-s, due to the following construction. Take all $n/2$-element subsets containing a fixed element f.

The odd case is non-trivial, but easy. It was independently discovered by many authors [17], [6], [7], [20].

THEOREM 1. *The maximum size of a family of pairwise qualitatively independent sets on n elements is*

$$\binom{n-1}{\lfloor \frac{n}{2} \rfloor - 1}.$$

The construction coincides with the even case.

[20] also contains good estimates on a more general problem. We say that $r \ge 2$ sets are qualitatively independent, if they divide X into 2^r non-empty sets. The maximum size of a family in which any r sets are qualitatively independent is estimated.

One may consider a more general condition. If all the sets in (1) are of size at least d then we say that A and B are *qualitatively independent of depth d*.

PROBLEM 1. What is the maximum size of a family of pairwise qualitatively independent sets of depth p on n elements?

It might be true that the obvious generalization of Theorem 1 holds for fixed d and large n. The case when $d = cn$ seems to be hard.

In what follows, we will consider another generalization. Before that a further motivation will be presented. It can be considered as the fourth source of the area of combinatorial search problems. Given n coins, one of them is counterfeit. It is known that the counterfeit coin is lighter than the good ones and it should be found by the minimum number of weighings using an equal arm balance. (No additional weights can be used.) The novelty in this problem is that one question (weighing) devides the groundset into three parts: the set of coins in the left arm, the set of coins in the right arm and the rest. In our earlier model the groundset was devided into two sets: the question set and its complement. This example of the equal arm balance suggests to introduce the notion of the *question partition*: $P = \{A_1, \dots, A_r\}$, where $A_1, \dots, A_r$ is a partition of the groundset X. The answer to this question determines the unique i satisfying $x \in A_i$. In this case a family $\mathcal{P}$ of partitions is given and the partitions for a linear search or tree search are chosen from this $\mathcal{P}$. If the number of parts in a partition does not exceed r we call it an *r-partition*.

The notion of the qualitatively independent partitions is straightforward. $\mathcal{P}_1$ and $\mathcal{P}_2$ are *qualitatively independent r-partitions* if they divide X into r^2 non-empty subsets. One cannot expect that the exact maximum number $m(n, r)$ of the pairwise qualitatively independent r-partitions could be determined, only its exponent. Poljak and Tuza [25] proved that

$$\limsup \frac{1}{n} \log m(n, r) \leq \frac{2}{r}.$$

A recent great achievement is

THEOREM 2 (Gargano, Körner, Vaccaro [13]).

$$\limsup \frac{1}{n} \log m(n, r) = \frac{2}{r}.$$

One should mention the preliminary work of Körner and Simonyi [22].

The following result does not belong to this section, but it is a surprising development in the area of the counterfeit coin problem and this is the best point to mention it. Suppose that n coins are given, $m \leq n$ of them are of weight $1 - \varepsilon$ (counterfeit coins) the rest of them are of weight 1 (good coins). Find the shortest tree search determining all the counterfeit coins. As the number of possibilities is $\binom{n}{m}$ and one question has three different answers, the minimum number of questions (in the worst case) is at least $\log_3 \binom{n}{m}$. This lower estimate is not the best because we cannot always divide the possibilities into 3 equal parts. The exact result, in general, expected to be quite complicated as it depends on number theoretical properties of n and m. The cases $m = 1, 2, 3, 4, 5$ are considered in [8], [32], [33], [34], [35].

It is surprising that the above lower estimate is still almost sharp in the following sense.

THEOREM 3 (Pyber [26]). *If exactly m (lighter) conterfeit coins are to be found among n coins then it can be done in at most*

$$\log_3 \binom{n}{m} + 15m$$

steps in all cases.

3. Optimal search with constraints on the sizes of question sets

Let us turn back to the original question. Suppose, again, that there is exactly one unknown element $x \in X$, the family of question sets is the family of all subsets in X and a tree search is used. Let k denote the length of the longest branch of the tree, that is, the number of questions in the worst case. Then the number of sequences of answers is at most 2^k since a sequence cannot be an extension of another one. For different x the sequence must be different. Hence $2^k \geq n$ holds. This implies $k \geq \lceil \log_2 n \rceil$. Now we show a construction of a (very special) tree search whose length is $\lceil \log_2 n \rceil$. It will be a linear search. Label the elements of X by $1, 2, \ldots, n$. These labels can be written with $\lceil \log_2 n \rceil$ binary digits. Let A_j consist of the elements of X whose label's jth digit is 1. The answers to these questions determine all digits of the label of the unknown x. Thus we can formulate the following theorem.

THEOREM 4 (Folklore). *The minimum number of questions in a linear (tree) search to find the only unknown element in an n-element set is*

$$\lceil \log_2 n \rceil .$$

Rényi asked what the situation is if $\mathcal{A} = \{A : A \in X, |A| \leq k\}$. The situation is considerably different here. *E.g.* the results for the tree search and the linear search do not coincide any more. The case of the tree search is easier both to describe and to prove.

Observe that any question set can be replaced by its complement. Thus, $k \leq n/2$ can be supposed. Furthermore, if after some questions in a tree search it is known that the unknown x is in a subset of size $s \leq 2k$ then x can be found by $\lceil \log_2 s \rceil$ further questions, using Theorem 4. Now let us give an optimal tree search. Write n in the form $n = qk + s$ where $k < s \leq 2k$ and take a partition $B_1, B_2, \ldots, B_q, B_{q+1}$ where $|B_1| = \ldots = |B_q| = k$, $|B_{q+1}| = s$. Ask $B_1, B_2, \ldots$ until the answer is "yes", $x \in B_i$ $(i \leq q)$. Then x can be found by $\lceil \log_2 k \rceil$ further questions. If the first such case is $i = q+1$ then we need $\lceil \log_2 s \rceil$ more questions. It is not hard to prove that this is (one of) the best tree search [16].

THEOREM 5. *Suppose that the question sets are the subsets of size at most $k \leq n$. Then the shortest tree search needs*

$$\left\lceil \frac{n}{k} \right\rceil - 2 + \left\lceil \log_2 \left(n - k \left\lceil \frac{n}{k} \right\rceil + 2k \right) \right\rceil .$$

When, in his seminar, Rényi asked what the minimum number of questions in a tree search is, many students (B. Bollobás, J. Galambos, T. Nemetz and D. Szász) brought the solution for the next seminar for the case $\frac{k(k+1)}{2} + 1 \leq n$. Then the result is $\lceil 2\frac{n-1}{k+1} \rceil$. The present author has constructed ([15]) the best linear search for all k. This leads to the following estimates.

THEOREM 6. *Denote by $l(n,k)$ the minimum length of a linear search finding an unknown element in an n-element set using question sets of size at most k. Then*

$$\frac{\log n}{\frac{k}{n}\log\frac{n}{k} + \frac{n-k}{n}\log\frac{n}{n-k}} \leq l(n,k) \leq \left\lceil \frac{\log 2n}{\log\frac{n}{k}} \right\rceil \cdot \frac{n}{k}.$$

(See also [23] and [37].) As it is pointed out by Dyachkov, the lower estimate is asymptotically sharp when $k = cn$.

Baranyai [5] generalized the construction for r-partitions whose parts (except the last one) are bounded in size. Proving this result he proved a "small lemma" which turned out to be a 120 year old conjecture of Sylvester:

THEOREM 7 (Baranyai [4]). *Suppose that k divides n. Then the set of all k-element subsets of the n-element set X can be partitioned into such classes that each class forms a partition of X.*

Although this famous result does not belong to the Combinatorial Search Theory, it was created under an indirect influence of Rényi. It is obvious to ask what can be said if k does not divide n. To formulate a conjecture concerning this general case, a new definition is needed. Suppose that the elements of the groundset X are ordered: $X = \{x_1, x_2, \dots, x_n\}$. Define $A_i = \{x_{(i-1)k+1}, x_{(i-1)k+2}, \dots, x_{(i-1)k+k}\}$ where the indices are considered mod n. The family $A_1, A_2, \dots, A_w$ where $w = n/\gcd(n,k)$ is called a *wreath*. (Neighboring k-element subsets are taken after each other until the end of one fits to the beginning of the first set.)

CONJECTURE (Baranyai and Katona). There are permutations of the groundset in such a way that these permutations of the wreath give all k-element sets exactly once.

It seems to be hard to settle this conjecture. Sylvester's conjecture was earlier attacked by algebraic methods and an algebraic way of thinking. Baranyai's brilliant idea was to use matrices and flows in networks. This conjecture, however, seems to be too algebraic. One does not expect to solve it without algebra. (Unless it is not true.)

Let us turn back to the search problems with restrictions on $\mathcal{A}$. We will use the problem of Steinhaus to obtain motivations. The problem actually became an important problem of computer science (with numbers rather than table tennis players). It is the simplest one of the so called sorting problems (see [21]). It is obvious by Theorem 4 that a tree search to find the actual permutation needs at least $\lceil \log_2 n! \rceil$ pairwise comparisons. This is $n \log_2 n + c_1 n + o(n)$. The tree search given by Ford and Johnson [12] (which is belived to be the best) needs $n \log_2 n + c_2 n + o(n)$ steps. That is, the first term is determined, but not the second one. Let us see the reason why the lower estimate $\lceil \log_2 n! \rceil$ cannot be realized by a tree search. To reach this bound we have to halve at each stage the set of possible cases, therefore the question sets should divide the groundset (of permutations) into 4 equal parts. This is, however not always possible. Consider the comparisons $a <?< b$ and $b <?< c$. Denote by A and B the set of permutations giving positive answer to the first and the second question, resp. Then two of the sets in (1) have size $n!/3$ and the other two have size $n!/6$. This is the motivation to the following investigations.

THEOREM 8 ([18].) *The minimum length of a linear search using question sets satisfying*

$$|A \cap B| \le 1$$

is

$$\left\lceil \frac{\sqrt{8n-7}-1}{2} \right\rceil.$$

Let us mention that this strange formula is equal to the smallest m such that

$$n \leq 1 + m + \binom{m}{2}.$$

[18] also gives the exact minimum up to an additional constant 1 for the case $|A \cap B| \leq 2$. Fairly good estimates are given for the cases $|A \cap B| \leq k$ where $k \leq c\sqrt[3]{n}$.

For the case of tree search let us start with an observation of Sebő [29]. If the first question set is A and the answer is $x \in A$ and the second question set is A_1 then it can be replaced by $A \cap A_1$. On the other hand, when $x \notin A$ then A_0 can be replaced by $A \cap A_0$. Continuing in this way we obtain a modified tree search where the question sets on different branches of the tree are disjoint while the ones along the same branch are in inclusion. Of course the lengths of the branches are unchanged. Thus, when looking for the shortest tree search, this strong property may be supposed. *E. g.* if $k = 1$ then the original condition becomes simply the condition that all question sets, with exception of the very first one, are of size 1.

Sebő [29] has determined the length of the shortest tree search under the condition $|A \cap B| \leq k$ for all $k < n/4$, however the formula is rather complicated so we give only the case $k = 1$ here.

THEOREM 9 (Sebő [29]). *The minimum length of a tree search using question sets satisfying*

$$|A \cap B| \leq 1$$

is

$$\left\lceil \frac{\sqrt{8n-7}-1}{2} \right\rceil.$$

Compare it with Theorem 8. The best linear search is not longer than the best tree search, in this case. For $k = 2$, however, the former one is about $\sqrt{3/2}$-times larger than the latter one.

[29] contains good estimates also for the case when the intersection of any m question sets is bounded.

The combination of the previous two types of constraints has not been studied, yet:

PROBLEM 2. Determine the length of the shortest linear and tree searchs, resp., under the conditions

$$|A| \leq k, \ |A \cap B| \leq l \quad \text{for all} \quad A, B \in \mathcal{A}.$$

Let us see the situation at the search of a permutation by binary comparisons. We observed that the comparisons $a <? < b$ and $b <? < c$ imply the existence of two question sets dividing the set of permutations into four parts containing one third, one third, one sixth, one sixth of the whole set, instead of the "good" proportion one fourth, one fourth, one fourth, one fourth. However, this is not true for all pairs of questions! What we can say is that among any $\frac{n}{2} + 1$ questions there is one such pair. One way of expressing the fact that two question sets are not intersecting each other in a "good" proportion is the use of the *entropy function* of the Information Theory. The *entropy of the partition* $A_1 \cup A_2 \cup A_3 \cup A_4$ of X is

$$\sum_{i=1}^{4} \frac{|A_i|}{|X|} \log_2 \frac{|X|}{|A_i|}.$$

This expression is 2 for the case when $A_i = \frac{1}{4}|X|$. It is known from Information Theory that it is smaller in all other cases. This suggests the following problem.

PROBLEM 3. Determine the length $f(t, E)$ of the shortest linear search under the condition that among any t question sets there is a pair such that the 4-partition induced by them has an entropy at most E.

We have only estimates. To formulate them some more definitions are needed. Put $h(x) = -x \log_2 x - (1 - x) \log_2(1 - x)$. The inverse of h is defined using the interval $0 \leq x \leq 1/2$ where it is monotone.

THEOREM 10.

$$\frac{2}{E} \log_2 n - (t - 1)\frac{2 - E}{E} \leq f(t, E) \leq \frac{2}{E} \log_2 n + O(\log_2 n).$$

PROOF. Start with the lower estimate. Let ξ be a random variable taking on values from X. Define the probabilities to be equal: $P(\xi = x) = 1/n$. Denote the question sets by $A_1, \ldots, A_m$. They define some further random variables:

$$\xi_i = \begin{cases} 0 & \text{if} \quad \xi \notin A_i \\ 1 & \text{if} \quad \xi \in A_i. \end{cases}$$

Now define the *entropy* of a random variable η taking on its different values with probabilities $p_1, \ldots, p_n$:

$$H(\eta) = -\sum_{i=1}^{n} p_i \log p_i.$$

Observe that ξ determines the random vector $(\xi_1, \ldots, \xi_m)$. On the other hand, as the answers to the questions "$x \in A_i$" determine x, therefore the converse is also true, $(\xi_1, \ldots, \xi_m)$ determines ξ. Consequently the distribution of the two random variables are identical and

$$H(\xi) = H((\xi_1, \ldots, \xi_m)) = \log_2 n. \tag{2}$$

Here we need an elementary lemma from Information Theory (see any textbook on Information Theory, *e.g.* [11], [9]):

$$H((\eta_1, \eta_2)) \le H(\eta_1) + H(\eta_2). \tag{3}$$

(2) and (3) imply

$$\log_2 n \le H((\xi_1, \xi_2)) + H(\xi_3) + H((\xi_4, \xi_7)) + \ldots \tag{4}$$

for any partition of the set $\{\xi_1, \ldots, \xi_m\}$ into one and two-element subsets. If they are all one-element sets then (4) leads to $\log_2 n \le m$, only, since the entropy of one ξ_i is bounded by one. However if we find M such disjoint pairs that $H((\xi_i, \xi_j)) \le E(< 2)$ then (4) results in

$$\log_2 n + M(2 - E) \le m. \tag{5}$$

To find the best M, the problem will be reformulated for graphs in an obvious way. Define a graph whose vertices are ξ_i and two vertices are connected iff $H((\xi_i, \xi_j)) \le E$. The following graph theoretic lemma is needed:

LEMMA. *Given a simple graph G on m vertices, there is at least one edge among any t vertices. Then it contains at least*

$$\left\lceil \frac{m - t + 1}{2} \right\rceil \tag{6}$$

vertex-disjoint edges in G. This result is sharp.

PROOF. Take the largest set L of vertex-disjoint edges. Let $|L| = l$. If $m - 2l \geq t$ then there are t vertices not contained in any member of L. By the conditions of the lemma there is an edge among these t edges which is vertex-disjoint to the members of L. This contradicts the maximality of L. The contradiction proves $m - 2l < t$ and the first part of the lemma.

Now consider the graph G of m vertices consisting of a complete graph on $m - t + 2$ vertices and isolated vertices. This graph obviously satisfies the conditions of the lemma and cannot contain more vertex-disjoint edges than (6). □

The lemma and (5) imply

$$\log_2 n + \frac{m - t + 1}{2}(2 - E) \leq m$$

and the lower estimate in Theorem 10.

The upper estimate will be proved by a simple construction. Define $k = \lfloor nh^{-1}(E/2) \rfloor$. Question sets of size at most k will be used. Then, by the monotonity of $h(x)$, we have $H(\xi_i) \leq h(k/n) \leq h(h^{-1}(E/2)) = E/2$. (3) implies $H(\xi_i, \xi_j) \leq E$, as needed.

Use the construction mentioned after Theorem 6 as k/n is a constant. Then the lower estimate is sharp in Theorem 6. It gives the upper estimate

$$\frac{\log n}{h(h^{-1}(E/2))} + O(\log n)$$

which coincides with the one given in the Theorem. □

Let us have some remarks concerning this Theorem.

1. It gives an approximate solution to the problem of Theorem 8 in a new case.

2. Problem 3 and Theorem 10 are not intended to help finding the shortest linear search for a permutation by pairwise comparisons. It is a trivial problem, one has to compare all $\binom{n}{2}$ pairs. However the solution of the analogous problem for the tree search might give a better lower estimate on the permutation problem.

4. Miscellany

As it was mentioned earlier, a tree search needs $n \log_2 n + O(n)$ steps to find the proper permutation of n objects (numbers) by pairwise comparisons. Modern computers have complex hardwares able to execute many operations simultanously. This is called parallel computation. As one object can be used only in one comparison at each moment, not more than $n/2$ parallel operations are possible. Therefore at least $O(\log n)$ steps are needed even if parallel steps are allowed. Ajtai, Komlós and Szemerédi [3] proved that this can really be done in this many steps.

Á. Varecza has proved many interesting results of sorting type. Let us mention only one of them here. Let $x_1, \ldots, x_{n-2}, y_1, y_2$ be distinct integers. It should be decided by pairwise comparisons if y_1 are y_2 neighboring in the natural order of all these numbers. It is easy to see that $2(n-2)$ steps are enough. However it is not at all trivial to prove that there is no shorter tree algorithm. It is proved in [36].

If there are more unknown elements in X then one question set A may give different answers. One of the natural models is when there are two possible answers: either a) there is one unknown element in A or b) there is none. Hwang and Vera Sós [14] gave good estimates for the minimum length of a linear search when the number d of unknown elements is known in advance.

J. N. Srivastava's following idea connected search theory with the theory of statistical factor analysis. The usual aim of factor analysis is (roughly speaking) to determine the weights of the influence of different factors for the investigated quantity. Now suppose that there are many possible factors, very few of them have a real influence the other ones have no influence (or are negligable). However it is not known which ones are the non-negligable. Thus in this model two things are done simultanously: a) determination of the non-negligable factors, b) determination of the weights of them. [19] contains results on the tools used to solve this problem, the so called *search designs*. These investigations led to the problem of finding the largest family S closed under the operation symmetric difference and such that $S - \{\emptyset\}$ is a Sperner family [19]. Generalizations can be found in [24].

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