

Comments for the author of 20.1073

1. The word orthonormed occurs several times and should be orthonormal
2. The word infact occurs several times and should be in fact
3. Quasi-orthogonal MASAS is used for a term that already exists in the literature, orthogonal MASAS (ref [23]). This should be changed
4. On p.4 line 2 consisiting should be consisting
5. After Lemma 3.1, a shorter proof of the main result, more in line with operator algebras and quantum channels, can be given as follows. It was noted before 3.1 that the ONB for  $\mathcal{B}$  can be taken to be self-adjoint. With this change,  $B_i = B_i^*$ , formula (12) becomes

$$\sum B_n X B_n = E_{\mathcal{B}}(X),$$

and denote the left hand side of this by  $\phi(X)$ . Then  $\phi$  is a trace preserving unital completely positive map. For any self-adjoint  $B \in \mathcal{B}$ , we have  $\phi(B) = B$  and  $\phi(B^2) \geq \phi(B)^2 = B^2$  from the theory of cp maps. Apply the trace to get equality, so that  $B$  is in the multiplicative domain of  $\phi$ . Then, for  $X \in \mathcal{B}$ ,  $\phi(XB) = \phi(X)\phi(B) = XB$ , showing that  $\mathcal{B}$  is a \*-algebra. If  $B' \in \mathcal{B}'$  then the formula for  $\phi$  shows that  $B' = \phi(B') \in \mathcal{B}$  so  $\mathcal{B}' \subseteq \mathcal{B}$ . Thus  $\mathcal{B}'$  is abelian and  $\mathcal{B}$  consists of a direct sum of matrix algebras which must be one dimensional otherwise the dimension  $d$  is exceeded. This shows that  $\mathcal{B}$  is a masa. Details on cp maps can be found in Paulsen's book on the subject.