REFEREE'S REPORT on LAA Submission #: 1002-082B

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Title: On quasi-orthogonal system of matrix algebras

This manuscript contains two factors:
(1) There is no decomposition by quasi-orthogonal subalgebras in a special case.
(2) A new tool to evaluate the existence of a system of quasi-orthogonal subalgebras.

This is a new approach to decompositions by quasi-orthogonal subalgebras. In (1), the considered subalgebras are very special. But the decompositions by such subalgebras relate to open problems.

The results obtained are interesting, and the whole proof written is comprehensible.

I have some questions and recommendations for this paper:

1. The term "quasi-orthogonal" is not good. In mathematics, "quasi-orthogonal" is equal to "orthogonal" (defined by Popa) and in mathematical physics, "quasi-orthogonal" is equal to "complementary". The term "quasi-orthogonal" is the newest word in the above three words and is used only a few papers. So the author should use the term "orthogonal" or "complementary".

2. The quantity $c(\mathcal{A}, \mathcal{B})$ is new one. But there is a similar quantity related to subfactor theory. Please check the paper "Angles between two subfactors" in J. Oper. Theory 32(1944), 209-241 written by Sano and Watatani and refer the paper.

3. In Sect 3, you prove that there are no decompositions of M_{n^2} into abelian subalgebras and 1 or 3 factors. Then it is natural to quess that there are no decompositions of M_{n^2} into abelian subalgebras and an odd number of facotrs. And this connects to the open problems (decompositions of $M_{(2n)^k}$ by M_{2n}). Please remark this in the paper.

4. In page 12, I can not understand what the example says.

(1) Unitary equivalence classes of $\mathcal{A}$ and $\mathcal{B}$ are $U\mathcal{A}U^*$ and $U\mathcal{B}U^*$ for some unitary U. $\mathcal{A}$ and $U\mathcal{B}U^*$ are not unitary equivalence classes of $\mathcal{A}$ and $\mathcal{B}$.

(2) Why can we say that $c(\mathcal{A}', \mathcal{B}')$ is not determined by $c(\mathcal{A}, \mathcal{B})$? What does the word "determine" mean ? If we can use the quantity $\dim(\mathcal{A} \cap \mathcal{B})$ for example, there is a possibility that $c(\mathcal{A}', \mathcal{B}')$ is determined by $c(\mathcal{A}, \mathcal{B})$ as in Theorem 4.2.

The following are minor corrections:
p.6, in example, the second trivially condition is satisfied only in the case $n = 3$.
p.7,l.-3, check "3 factors and 2 MASA's".
p.9, in (18), $W_2 \to A_2$.
p.10,l.-10, "previous lemma" $\to$ "Lemma 3.4".
p.10,l.-5, $B_1 \to B_2$.
p.11, in (23), add "-".

p.11,l.6, $\mathcal{B}_1 \to B_1$.

p.11,l.20, $\mathcal{B}_1 \to \mathcal{B}'_1$.

p.12, in footnote, "are" exists twice.

p.12,l.18, remove "Finally".

p.12,l.-2, check the equation.

p.13, in the fifth condition, homogeneity is "not" a condition about isomorphism class. Does "sits" mean the unitary equivalence ?

p.13, in Lemma 4.1, $E_{\mathcal{A}} \to E_{\mathcal{A}'}$.

p.13, in Theorem 4.2, $\frac{n}{N} \to \frac{N}{n}$.

p.14, in (33), check ")".

p.15,l.-14, add ",".

p.15,l.-6, remove "not only".