

Description of MAP exams

The duration of a MAP exam is always 2 hours. The total number of points that can be obtained for a problem is always indicated in parenthesis after the text of the problem. You may encounter four type of problems, marked with letters E, R, RP, or S:

- E** (*Explain.*) At this type you should explain the significance of a mathematical fact or theorem. Example: *Contrary to the quantum case, why can "uncertainty" in classical probability theory always considered to be a consequence of lack of information?* However, this type will not appear at the midterm exam (only at the final one).
- R** (*Recall.*) At this type you should simply recall a definition or a theorem, or list some learnt properties of a mathematical object, but you are *not* asked to give any proofs. Example: *Define what a lattice is, and define what do we call a not-function on a lattice.* Or: *State the spectral theorem (in finite dimensions).* Or: *List the defining properties of a scalar product.*
- RP** (*ReProve.*) At this type you should reproduce the proof given in class of a theorem. You do not have to remember all proofs; you are previously given a smaller list of theorems (see the list later), the RP question at exam will be *exactly* one of them.
- S** (*Solve.*) At this type you have to resolve a problem. You are asked to give rigorous mathematical arguments, which you should carefully explain. One of the problems of this type will come from a previous exercise sheet (in the mock exam S1 is such, whereas S2 is "new").

List of possible RP questions for the midterm exam

- Prove that if \mathcal{L} is a distributive ortho-lattice, then a pure probability law $p : \mathcal{L} \rightarrow [0, 1]$ is automatically dispersion free.
- Define what a projection is, and prove that it can be characterized by the algebraic relation $P^2 = P$.

- Prove the existence and uniqueness of the adjoint (in finite dimensions).
- Prove (in finite dimensions) that if N is a normal operator, then λ is an eigenvalue for N if and only if $\bar{\lambda}$ is an eigenvalue for $\text{Sp}(N^*)$, and moreover, that the eigenspace of N corresponding to λ coincides with the eigenspace of N^* corresponding to $\bar{\lambda}$. Conclude that the eigenspaces of N are pairwise orthogonal.

Midterm mock exam

R1. Define what a pure probability law is. (6p)

R2. List all properties (without proof) of the adjoint (with respect to addition, multiplication, image, nucleus ect.) learnt in class. (6p)

RP. Define what a projection is, and prove that it can be characterized by the algebraic relation $P^2 = P$. (12p)

S1. Let $A, B \in \mathcal{B}(\mathcal{H})$ two self-adjoint operators. Show that both $\text{Tr}(A^2B^2)$ and $\text{Tr}((AB)^2)$ are reals,

$$\text{Tr}(A^2B^2) \geq \text{Tr}((AB)^2),$$

and equality holds if and only if $AB = BA$. (12p)

S2. Calculate, using the spectral decomposition, the n^{th} power of the matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. (12p)