MAP: HOMEWORK #2

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Note: my LaTeX is being stupid, and not letting me type integrals, so instead, assume that $\mathcal{L}$is an integral–I'll try to fix that later, so sorry!

Problem 2.1

The functions need to be continuous, because otherwise the problem would be trivial, as equality could hold if one of the functions had an integral of zero (as a noncontinuous function which is zero everywhere and 1 at only one point, while being integrable, and nonzero can have an integral equalling zero). Likewise, g needs to be strictly positive (and continuous), because otherwise g could be zero and therefore have an integral of zero, and equality would hold.

$L_{[0,1]}g\left|f\right|^2 L_{[0,1]}g\left|h\right|^2 = L_{[0,1]}g^2 L_{[0,1]}\left|f\right|^2 L_{[0,1]}\left|h\right|^2$ and this is equal to

$L_{[0,1]}\left|g\right|^2 L_{[0,1]}\left|f\right|^2 L_{[0,1]}\left|h\right|^2$ because g is strictly positive. However, it is only possible that this is equal to $L_{[0,1]}\left|gfh\right|^2 = \left|L_{[0,1]}gfh\right|^2$ only if h is a multiple of f because otherwise function composition would not be commutative (for example, consider $f = x^2$and $h = x + 1$. Then the integrals of $\left|f\right|^2$ and $\left|h\right|^2$ multiplied together would not equal the integral of $\left|fh\right|^2$), and therefore equality would not hold.

Problem 2.2

Part 1. If Alice's missing number is 5 then we know that at most one of Bob's numbers is 4, but besides the remaining number(s) not equalling 4, it is not possible to determine what Bob's number would be. Although we know that the rank of both matrices would be 2, Bob's extra number would still be arbitrary.

If Alice's missing number is 6, then we know that both of Bob's numbers would be 4, because then the ranks would both be one. 4 is the only choice if we are to maintain the existence of only one independent basis.

Part 2. If Alice's missing number is six, then Bob's missing numbers are 4 and 4. Alice's matrix is $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$ and Bob's matrix is $\begin{pmatrix} 1 & 1 & 1 \\ 4 & 4 & 4 \end{pmatrix}$. If we could find a vector which would be mapped to the kernel of one, but not the other, then we would be done. Such a vector exists, and that vector is $(0, 2, -3)$, which would be mapped to the kernel of Alice's matrix, but it is mapped to $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ in Bob's matrix.