Lemma 2 *Let C be a convex cone in a vector space and $F : C \to \mathbb{R}$ be a convex function such that*

$$\lim_{x \to +0} x^{-1}(F(A + xB) - F(A)) \equiv G(A, B)$$

exists for all $A, B \in C$ and

$$F(\lambda A) = \lambda F(A)$$

for every $\lambda > 0$ and $A \in C$. Then $F(B) \geq G(A, B)$.

Proof. The homogeneity and convexity of F yield

$$F(A + xB) = (1 + x)F((1 + x)^{-1}A + x(1 + x)^{-1}B) \leq F(A) + xF(B)\,.$$

Now subtract $F(A)$ from both sides of the inequality, divide it by $x > 0$ and take the limit $x \to +0$. □