EXTREMAL GRAPH PROBLEMS

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<u>Notations</u>. $v(G)$, $e(G)$, $\chi(G)$ denote the number of vertices, edges and the chromatic number of the graph G. Here the graphs have no directed, multiple or loop edges. $\overset{d}{\underset{i=1}{\times}} G_i$ denotes the product of graphs G_i, i.e. the graph, obtained by joining vertices of G_i to the vertices of the other G_i-s.

Generalizing a well-known theorem of Turán [1] Erdös and I have proved independently [3], [4] that for any given graph $M_1,\dots,M_k$ and fixed n if K^n has maximum number of edges among graphs of n vertices, not containing any M_i as a subgraph, then

<u>Theorem A</u>. *There exist graphs $N_1,\dots,N_d$, $(d+1 = \min \chi(M_i))$ such that K^n can be obtained from $\overset{d}{\underset{i=1}{\times}} N_i$ omitting $0(n^{2-\frac{1}{r}})$ edges from it. Here is an integer depending only on $M_1,\dots,M_\mu$ and*

(1) $v(N_i) = \frac{n}{d} + 0(n^{1-\frac{1}{r}})$, $e(N_i) = 0(n^{2-\frac{1}{r}})$

(2) *any vertex of N_i has valence* $\geq \frac{n}{d}(d-1) + 0(n^{1-\frac{1}{r}})$

(3) *the number of vertices of N_i joined to at least εn vertices of the same N_i is $0_\varepsilon(1)$.*

The graph K^n is called the *extremal graph* for $M_1,\dots,M_\mu$. Theorem A shows that the extremal graphs for $M_1,\dots,M_\mu$ are fairly well determined by $\min \chi(M_i)$, they depend loosely on the structure of M_i-s.

How the structure of M_i-s influence the structure of the extremal graphs? Erdös and I have proved [5] that the extremal graphs for $K(3,r_1,\dots,r_d)$ are products: $K^a = \overset{d}{\underset{i=1}{\times}} N_i$ where $3 \leq r_1 \leq r_d$ and