CHARACTER EXPANSIVENESS IN FINITE GROUPS

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Abstract. We say that a finite group G is conjugacy expansive if for any normal subset S and any conjugacy class C of G the normal set SC consists of at least as many conjugacy classes of G as S does. Halasi, Maróti, Sidki, Bezerra have shown that a group is conjugacy expansive if and only if it is a direct product of conjugacy expansive simple or abelian groups.

By considering a character analogue of the above, we say that a finite group G is character expansive if for any complex character α and irreducible character χ of G the character $\alpha\chi$ has at least as many irreducible constituents, counting without multiplicity, as α does. In this paper we take some initial steps in determining character expansive groups.

1. Introduction

The product of two conjugacy classes in a finite group usually consists of many conjugacy classes. In [8] a finite group G was called *(conjugacy) expansive* if for any normal subset S and any conjugacy class C of G the normal set SC consists of at least as many conjugacy classes of G as S does. It has been proved in the same paper that G is conjugacy expansive if and only if it is the direct product of conjugacy expansive simple or abelian groups. Hence, to classify such groups it is sufficient to determine which simple groups are conjugacy expansive. It is conjectured that all simple groups are such groups. In fact, the groups $\mathrm{L}_2(q)$ and $\mathrm{Suz}(q)$ are all conjugacy expansive when simple and the 138 non-abelian finite simple groups whose character table can be found in the Gap [5] character table library are also conjugacy expansive [8].

In this paper the character analogue of conjugacy expansiveness is considered. We say that a finite group G is *character expansive* if for any complex character α and irreducible character χ the number of irreducible constituents of the product $\alpha\chi$ (counting without multiplicity) is at least the number of irreducible constituents of α, again counting without multiplicity. For example, abelian groups are character expansive.

Our first observations on character expansive groups are the following.

Theorem 1.1. *For a character expansive group G we have the following.*

(1) *If G is solvable then it is abelian.*

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(2) *If G is almost simple then it is simple.*
(3) *If G is quasisimple then it is simple.*

The ideas of [8, Section 3] can directly be translated to this character context to prove the following.

Theorem 1.2. *Let G be a direct product of groups. Then G is character expansive if and only if every direct factor of G is character expansive.*

Theorems 1.1, 1.2, and the results on conjugacy classes above suggest us to consider the following.

Problem 1.3. *Is it true that a character expansive group is a direct product of simple or abelian groups?*

The converse of Problem 1.3 is false. For let $n = k^2$ for some integer k at least 3. By [1, Theorem 5.6], there are four irreducible characters χ_1, $\chi_2 \neq \overline{\chi}_2$, χ_3 of A_n so that $\chi_1\chi_2 = \chi_3 = \chi_1\overline{\chi}_2$. This means that A_n cannot be character expansive for $n = k^2$. Furthermore, for the same reason, none of the sporadic simple groups Co_1, Co_2, Co_3, Fi_{24}', M, M_{12}, M_{24}, and Th can be character expansive. (Using the Hungarian algorithm [9] it can be shown by computer that among the 138 non-abelian simple groups in the Gap [5] library all other groups are character expansive.)

Unfortunately we are unable to solve Problem 1.3. We can only show

Theorem 1.4. *A minimal counterexample to Problem 1.3 has a unique minimal normal subgroup and that is abelian and non-central.*

Let G be a group which is the direct product of non-abelian finite simple groups and let V be a finite faithful irreducible FG-module for some prime field F. For a complex linear character λ of V let $I_G(\lambda)$ be the stabilizer of λ in G and for a finite group H let $k(H)$ be the number of conjugacy classes of H. An affirmative answer to the following problem would imply Problem 1.3.

Problem 1.5. *With the notations and assumptions above, does there exists $\lambda \in \mathrm{Irr}(V)$ with $k(I_G(\lambda)) < k(G)$?*

Interestingly, Problem 1.5 seems to be close to the $k(GV)$ problem.

Theorem 1.6. *With the notations and assumptions above, Problem 1.5 has an affirmative solution if G is simple and $(|G|, |V|) = 1$, or if $G = \mathrm{GL}(n, 2)$ and $|V| = 2^n$ with $n \geq 3$.*

2. Basic results

The paper [8] considered the following "weaker" notion than conjugacy expansiveness. We say that G is *normal conjugacy expansive* if for any normal subgroup N and any conjugacy class C of G the normal set NC consists of at least as many conjugacy classes of G as N does.

Lemma 2.1. *For a finite group G the following are equivalent.*

(1) *G is normal conjugacy expansive;*
(2) *G is a direct product of simple or abelian groups;*
(3) *$k(G) = k(N)k(G/N)$ for all normal subgroups N of G.*

Proof. This is part of [8, Theorem 1.1]. □

In this note we try to find character analogues of the results in [8]. For this we define $n(\alpha)$ to be the number of irreducible constituents, counting without multiplicity, of a character α of G. So G is character expansive if for any character α and any irreducible character χ we have $n(\alpha) \leq n(\alpha\chi)$. Furthermore we say that G is *normal character expansive* if for any normal subgroup N and any irreducible character χ of G we have $n(1_N^G) \leq n(1_N^G \cdot \chi)$. Here $n(1_N^G)$ is clearly $k(G/N)$. Also, character expansiveness implies normal character expansiveness.

We hope to show that a group is normal character expansive if and only if it is a direct product of simple or abelian groups. Our first observation is the following.

Lemma 2.2. *Every factor group of a normal character expansive group is normal character expansive.*

Proof. This essentially follows from the correspondence theorem about normal subgroups in quotient groups. □

Note that we do not know whether a normal subgroup of a normal character expansive group is normal character expansive.

Lemma 2.3. *Let N be a normal subgroup of a finite group G such that $k(G/N) \leq n(1_N^G \cdot \chi)$ for any irreducible character χ of G. Then the number of irreducible characters of G lying above any irreducible character of N is at least $k(G/N)$.*

Proof. Let θ be an arbitrary irreducible character of N and let χ be an irreducible character of G lying above θ. Now, for an arbitrary irreducible character ψ of G, we have $\langle 1_N^G \cdot \chi, \psi \rangle = \langle \chi_N, \psi_N \rangle$ which is non-zero if and only if ψ lies above θ. Thus $n(1_N^G \cdot \chi)$ is equal to the number of irreducible characters of G lying above θ. The result follows. □

Lemma 2.4. *Let N be a normal subgroup of a finite group G. Put $H = G/N$. Then the number of irreducible characters of G lying above a fixed irreducible character θ of N is at most $k(I_H(\theta))$. Hence if G is normal character expansive then $k(H) \leq k(I_H(\theta))$. Furthermore if we also have that H is abelian then θ is G-invariant.*

Proof. The first statement is [4, Corollary, Page 178]. The second follows from Lemma 2.3. For the third statement notice that $|H| = k(H) \leq k(I_H(\theta)) \leq |H|$. □

Finally, we will need an old result of Nagao [12, Lemma 1].

Lemma 2.5. *Let N be a normal subgroup of a finite group G. Then $k(G) \leq k(N)k(G/N)$.*

3. Proof of Theorem 1.1

The next three lemmas give all of Theorem 1.1.

Lemma 3.1. *A solvable normal character expansive group is abelian.*

Proof. Let G be a minimal counterexample to the statement of the lemma and let N be a non-trivial normal subgroup of G. By Lemma 2.2 we know that G/N is normal character expansive and so, by the minimality of G, we have that $H = G/N$ is abelian. Let θ be an arbitrary irreducible character of N. Then θ is G-invariant by Lemma 2.4. Hence, by Lemma 2.3, we have $k(N)k(G/N) \leq k(G)$. On the other hand, by Lemma 2.5, we see that $k(G) \leq k(N)k(G/N)$. We conclude that for *any* normal subgroup N of G we have $k(G) = k(N)k(G/N)$. Finally, apply Lemma 2.1 to conclude that G is abelian. □

Lemma 3.2. *An almost simple normal character expansive group is simple.*

Proof. Let G be an almost simple normal character expansive group with socle N. Then the factor group G/N is normal character expansive by Lemma 2.2. Moreover G/N is solvable by Schreier's conjecture. Thus G/N is abelian by Lemma 3.1. Thus every irreducible character of N is G-invariant, by Lemma 2.4. By Brauer's permutation lemma, this means that every conjugacy class of N is G-invariant. But, if G/N is non-trivial, this contradicts [2, Theorem C] which states that any outer automorphism of a non-abelian finite simple group N fuses some of the conjugacy classes of N. □

Lemma 3.3. *A quasisimple normal character expansive group is simple.*

Proof. Let G be a quasisimple normal character expansive group. Let Z be its center. Then every proper normal subgroup N of G is contained in Z. Hence every irreducible character of N is G-invariant. By Lemma 2.3, we have $k(N)k(G/N) \leq k(G)$ which forces $k(G) = k(N)k(G/N)$ using Lemma 2.5. Now apply Lemma 2.1 to conclude that G is simple. □

4. Proof of Theorem 1.4

In order to prove Theorem 1.4, our first step is to show that any minimal normal subgroup of a minimal counterexample to Problem 1.3 is abelian. To achieve this, we need two lemmas.

Lemma 4.1. *Let G be a primitive permutation group on a finite set Ω. Then there exists a subset Δ of Ω with $k(G_\Delta) < k(G)$. In fact, Δ can be chosen so that $|\Delta| = 1$ or $G_\Delta = 1$.*

Proof. Put $n = |\Omega|$. We may assume that $n \geq 3$. Since $k(A_{n-1}) < k(A_n)$ and $k(S_{n-1}) < k(S_n)$, we may also assume that G does not contain A_n.

By [14, Theorem 2], there exists a subset Δ of Ω with $G_\Delta = 1$ unless G is a member of an explicit list of 43 permutation groups (of degrees at most 32). Using Gap [5] it can be checked that for all these exceptional groups G we have $k(G_\Delta) < k(G)$ whenever $|\Delta| = 1$. □

Lemma 4.2. *For any non-abelian finite simple group G there exists a non-trivial irreducible character of G stabilized by* $\mathrm{Aut}(G)$.

Proof. If G is a finite simple group of Lie type then such a character can be taken to be the Steinberg character [7, Theorem A]. Otherwise we may assume that $|\mathrm{Out}(G)| = 2$. Assume the claim is false. Then $\sum_{1\neq\chi\in\mathrm{Irr}(G)} \chi(1)^2 = |G| - 1$ is even which is a contradiction. □

Proposition 4.3. *Let G be a minimal counterexample to Problem 1.3. Then every minimal normal subgroup of G is abelian.*

Proof. Let G be as in the statement of the proposition. Then every proper factor group of G is a direct product of simple or abelian groups. Suppose for a contradiction that $N \cong S^\ell$ is a minimal normal subgroup of G for some non-abelian simple group S and some positive integer ℓ.

Suppose $\ell = 1$ and let $M = S \times C_G(S)$. Then $G/C_G(S)$ is almost simple and normal character expansive. Hence, by Lemma 3.2, it is simple. So $G = M$ which is a contradiction. Thus $\ell > 1$.

The group G/N acts transitively (but not necessarily faithfully) on the set Σ of simple factors of N. Clearly $|\Sigma| = \ell > 1$. We may naturally think of N as $S_1 \times \cdots \times S_\ell$ with each S_i isomorphic to S. Let Ω be a system of maximal blocks for G/N acting on Σ; the group G/N acts primitively on Ω. Let the kernel of the action of G/N on Ω be K/N.

Now let Δ be a subset of Ω with $k((G/K)_\Delta) < k(G/K)$. Such a set exists by Lemma 4.1.

Let χ be a non-trivial complex irreducible character of S stabilized by $\mathrm{Aut}(S)$. Such a character exists by Lemma 4.2. Now consider the irreducible character $\psi = \otimes_{i=1}^{\ell} \psi_i$ of N where $\psi_i \in \mathrm{Irr}(S_i)$ for every i in $\{1, \ldots, \ell\}$ with $\psi_i = \chi$ if S_i is an element of an element of Δ and $\psi_i = 1$ otherwise. By our construction $I_G(\psi) \geq K$ and $I_G(\psi)/K = (G/K)_\Delta$.

Then $k(G/N) = k(G/K)k(K/N) > k((G/K)_\Delta)k(K/N) \geq k(I_{G/N}(\psi))$ where the last inequality follows from Lemma 2.5. This is a contradiction to Lemma 2.4. □

Lemma 4.4. *Let G be a minimal counterexample to Problem 1.3. Then G has a unique minimal normal subgroup and that is abelian.*

Proof. By Proposition 4.3 every minimal normal subgroup of G is abelian. So for a contradiction assume that N_1 and N_2 are two minimal (abelian) normal subgroups of G. Note that G is non-solvable by Lemma 3.1. By the minimality of G and by Lemma 2.2, we have $\overline{G} = G/N_1 = \overline{S}_1 \times \cdots \times \overline{S}_m \times \overline{A}$ for some non-abelian simple groups $\overline{S}_1, \ldots, \overline{S}_m$ and abelian group $\overline{A}$. Now fix an index i with $1 \leq i \leq m$. Let S_i be the preimage of $\overline{S}_i$ in G. Then S_i is normal in G. Now again, $\overline{\overline{G}} = G/N_2 = H_1 \times \cdots \times H_m \times B$ for some H_j's with $H_j \cong \overline{S}_j$ with $1 \leq j \leq m$ and abelian group B. Since $S_i \cap N_2 = 1$, we have $\overline{\overline{S}}_i \cong S_i$ for the image $\overline{\overline{S}}_i$ of S_i in $\overline{\overline{G}} = G/N_2$. Since $\overline{\overline{S}}_i$ is normal in $\overline{\overline{G}}$, it must be a direct product of simple and abelian groups. Hence $S_i = U_i \times N_1$ for some $U_i \cong \overline{S}_i$. This implies that U_i is a non-abelian minimal normal subgroup in G. This contradicts Proposition 4.3.

□

Lemma 4.5. *Let G be a minimal counterexample to Problem 1.3. Then the unique minimal normal subgroup of G is not central.*

Proof. Let N be the unique (abelian) minimal normal subgroup of G (Lemma 4.4). (Again, G is non-solvable by Lemma 3.1.) Suppose for a contradiction that $N \leq Z(G)$ (and is of prime order). Since G is normal character expansive, we have $k(N)k(G/N) \leq k(G)$ by Lemma 2.4. But then $k(G) = k(N)k(G/N)$ by Lemma 2.5. By the minimality of G and Lemma 2.2, $G/N = T \times A$ for some group T which is a direct product of non-abelian finite simple groups and some abelian group A. For a simple direct factor $\overline{S}$ of T let S be the preimage of $\overline{S}$ in G. Clearly, S is normal in G. If no S is quasisimple then we arrive to a contradiction as in Lemma 4.4. So assume that a given S is quasisimple. By repeated use of Lemma 2.5 and noting that equality in Lemma 2.5 occurs for direct products, we have

$$k(\overline{S})k(G/S)k(N) = k(G/N)k(N) = k(G) \leq k(S)k(G/S)$$

which gives $k(\overline{S})k(N) \leq k(S)$. Hence, again by Lemma 2.5, $k(S) = k(\overline{S})k(N)$. But this contradicts Lemma 2.1. □

This finishes the proof of Theorem 1.4.

5. Problem 1.5 implies Problem 1.3

Let H be a minimal counterexample to Problem 1.3. By Theorem 1.4 the group H has a unique minimal normal subgroup V and this is abelian and non-central. Thus V is (a non-trivial) irreducible G-module where $G = H/V$ is a direct product of simple or abelian groups (Lemma 2.2). By Lemma 2.4, we have $k(G) \leq k(I_G(\lambda))$ for every $\lambda \in \mathrm{Irr}(V)$.

Let us call a pair (V, G) with the above properties bad, allowing V to be a completely reducible G-module. We will show that an affirmative answer to Problem 1.5 implies that bad pairs do not exist.

So suppose that (V, G) is a bad pair with $|G| + |V|$ minimal.

Suppose that $G = T \times A$ where T is a group which is a direct product of non-abelian finite simple groups and A is abelian. (T is non-trivial by part (1) of Theorem 1.1.) We have that

$$I_G(\lambda)T/T \cong I_G(\lambda)/(I_G(\lambda) \cap T) = I_G(\lambda)/I_T(\lambda)$$

is an abelian group of order at most $|A|$ for any $\lambda \in \mathrm{Irr}(V)$. Hence

$$|A| \cdot k(T) = k(G) \leq k(I_G(\lambda)) \leq k(I_T(\lambda)) \cdot |A|$$

for any $\lambda \in \mathrm{Irr}(V)$ where the last inequality follows from Lemma 2.5. Since V is a completely reducible T-module, the pair (V, T) is bad, hence we may assume that $G = T$.

Let W be a non-trivial irreducible G-submodule of V and let U be a submodule of V complementing W. Consider the irreducible characters $\lambda = \lambda_W \otimes 1_U$ of V where λ_W runs through the set of irreducible characters of W and 1_U is the trivial character of U. We clearly have $k(G) \leq k(I_G(\lambda)) = k(I_G(\lambda_W))$. This means that the pair (W, G) is bad. Hence $V = W$ and we assume from now on that V is an irreducible G-module.

Let $M = C_G(V)$. Then

$$k(G/M)k(M) = k(G) \leq k(I_G(\lambda)) \leq k(I_G(\lambda)/M)k(M)$$

implies $k(G/M) \leq k(I_G(\lambda)/M)$ for every $\lambda \in \mathrm{Irr}(V)$. This means that the pair $(V, G/M)$ is bad. Hence $M = 1$ and we may assume that V is a faithful G-module.

But such a pair (V, G) cannot exist by Problem 1.5.

6. Proof of Theorem 1.6

Let V be a finite faithful irreducible FG-module for a prime field F and a non-abelian finite simple group G.

Suppose first that $(|G|, |V|) = 1$. Since the action is coprime, $\mathrm{Irr}(V)$ and V are isomorphic G-sets. Hence, to prove Theorem 1.6 in this case, it is sufficient to find a vector v in V with $k(C_G(v)) < k(G)$.

If G has a regular orbit on V then there is nothing to show. If $G = A_n$ (with $n \geq 5$) and V is the deleted permutation module coming from the permutation module with permutation basis $\{e_1, \ldots, e_n\}$ then $C_{A_n}(e_1 - e_2) = A_{n-2}$ and $k(A_{n-2}) < k(A_n)$. Otherwise, if G has no regular orbit on V and V is not a deleted permutation module then (V, G) belongs to a finite list of examples as in the table following [6, Theorem 2.2]. There information can be found about H, a subgroup of smallest possible size with $H = C_G(v)$ for some $v \in V$. It can readily be checked that $k(H) < k(G)$ in all cases.

This proves the first part of Theorem 1.6.

Now suppose that $G = \mathrm{GL}(n,2)$ and $|V| = 2^n$ with $n \geq 3$. Here the action is no longer coprime, but by Brauer's permutation lemma we know that G has exactly two orbits on $\mathrm{Irr}(V)$. By [13, Example 13.1 (ii)], we have $I_G(\lambda) \cong \mathrm{AGL}(n-1,2)$ for every non-trivial $\lambda \in \mathrm{Irr}(V)$. So if c_r denotes $k(\mathrm{GL}(r,2))$ for $r \geq 1$ and $c_0 = 1$ then $k(I_G(\lambda)) = \sum_{r=0}^{n-1} c_r$. Hence the inequality $\sum_{r=0}^{n-1} c_r < c_n$ must be shown for all $n \geq 3$. This is true for $n \leq 50$ by Gap [5]. So assume that $n > 50$.

For non-negative integers s and n let $p(s,n)$ be the number of partitions of n with at most s parts. For a non-negative integer m put

$$c_{n,m} = \underbrace{\sum_{s=0}^{\infty}\sum_{j=0}^{\infty}}_{n=(s+1)(m+s/2)+j} (-1)^s p(s,j).$$

Also, put $a_n = 2^n - (2^{[(n-1)/2]} + 2^{[(n-1)/2]-1} + \cdots + 2^{[n/3]})$ and $b_n = \sum_{m=0}^{[n/3]-1} c_{n,m} 2^m$. Then it can be derived from [10, Pages 28-29] that

$$c_n = a_n + b_n.$$

Next we will bound $c_{n,m}$ in various important cases.

If $[n/4] \leq m \leq [n/3]-1$ then $0 \leq c_{n,m} \leq n/8 + 1$.

If $[n/5] \leq m \leq [n/4]-1$ then $|c_{n,m}| \leq n^2/50 + n/10 + 2$.

Moreover for any integer $k \geq 8$ with $1 \leq [n/k] \leq m \leq [n/(k-1)]-1$ we have

$$|c_{n,m}| \leq n^{k-3}/(2 \cdot (k-2)!).$$

Furthermore, in general, $|c_{n,m}| \leq n \cdot p(n)$ where $p(n)$ denotes the number of partitions of n. For $p(n)$ we have the explicit upper bound $p(n) \leq e^{\pi\sqrt{2n/3}}$ found in [3] and also the exact values of $p(n)$ for $n \leq 150$ from [5]. Using these we can show the following lemma for $n \geq 100$.

Lemma 6.1. *We have* $2^n - 1.5 \cdot 2^{n/2} \leq c_n$ *for all* n *and* $c_n \leq 2^n - 0.63 \cdot 2^{n/2}$ *for* $n > 8$.

Proof. By the above this is true for $n \geq 500$. For smaller values of n the inequalities can be checked by Mathematica [11]. □

In order to complete the proof (of the second part) of Theorem 1.6 we need to show that $\sum_{i=0}^{n-1} c_i < c_n$ holds (for $n > 50$). It can be checked that

$$\sum_{i=0}^{8} c_i \leq 2 + \sum_{i=0}^{8}(2^i - 0.63 \cdot 2^{i/2}).$$

Thus we have $\sum_{i=0}^{n-1} c_i \leq 2 + \sum_{i=0}^{n-1}(2^i - 0.63 \cdot 2^{i/2})$ by Lemma 6.1. This and Lemma 6.1 imply that

$$\sum_{i=0}^{n-1} c_i \leq 2^n + 1 - 0.63 \cdot \frac{2^{n/2}-1}{\sqrt{2}-1} < 2^n - 1.5 \cdot 2^{n/2} < c_n.$$

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