Analytic Number Theory WS14/15
Homework 3 (Due 9.01.2015)

December 15, 2014

Problem 1.(Orthogonality relations)

i. Let q be an odd prime. Prove that

$$\frac{1}{q}\sum_{\ell=0}^{q-1} e^{2\pi i k\ell/q} = \begin{cases} 1 & \text{if } k \equiv 0 \pmod q\,; \\ 0 & \text{otherwise.} \end{cases}$$

ii. Prove that for any integer k

$$\int_0^1 e^{2\pi i k x}dx = \begin{cases} 1 & \text{if } k = 0\,; \\ 0 & \text{otherwise.} \end{cases}$$

Problem 2. Let as usual p denotes a prime number and

$$P(n) = \prod_{\substack{p\mid n \\ p>\log n}} \left(1 - \frac{1}{p}\right) .$$

Prove that $\lim_{n\to\infty} P(n) = 1$.
(Hint: It might help to take logarithm on both sides and use an upper bound for the number of prime divisors of n, which are $> \log n$.)

Problem 3.(Mertens formula)

i. Prove that for some constant C

$$\prod_{p\le x}\left(1 - \frac{1}{p}\right) = \frac{e^{-C}}{\log x}\left(1 + \mathcal{O}\left(\frac{1}{\log x}\right)\right) .$$

ii*. Prove that $C = \gamma$, where γ is the Euler constant.