

Analytic Number Theory WS14/15
Homework 2 (Due 21.11.2014)

November 7, 2014

Problem 1. Prove the identity

$$\sum_{n=1}^{\infty} \frac{\varphi(n)}{n^s} = \frac{\zeta(s-1)}{\zeta(s)}, \operatorname{Re}(s) > 2.$$

Problem 2. Prove the asymptotic formula

$$\sum_{k=n}^{\infty} \frac{\varphi(k)}{k^3} = \frac{6}{\pi^2 n} + \mathcal{O}\left(\frac{\log n}{n^2}\right).$$

Problem 3*. Use the Dirichlet hyperbola method to show that

$$\sum_{n \leq x} \tau(n^2 + 1) = \frac{3}{\pi} x \log x + \mathcal{O}(x).$$