Europ. J. Combinatorics (1991) **12**, 483–489

Covering t-element Sets by Partitions

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Partitions of a set V form a t-cover if each t-element subset is covered by some block of some partitions. The rank of a t-cover is the size of the largest block appearing. What is the minimum rank of a t-cover of an n-element set, consisting of r partitions? The main result says that it is at least n/q, where q is the smallest integer satisfying $r \leq q^{t-1} + q^{t-2} + \cdots + q + 1$.

1. INTRODUCTION

A *partition* is a decomposition of a set into pairwise disjoint subsets, called *blocks*. Partitions of a set V define a *t-cover* if each t-element subset of V is covered by at least one block. The *rank* of a t-cover is the cardinality of the largest block appearing in the partitions. Define $f(n, r, t)$ as the minimum rank of a t-cover of an n-element set with r partitions. Thus a t-cover is a relaxation of resolvable t-designs with r parallel classes. The same function can be defined by the following Ramsey-type problem: $f(n, r, t)$ is the maximum m such that in any r-coloring of the edges of $\mathbf{K}_t^n$ (the complete t-uniform hypergraph on n vertices) there exists a monochromatic connected component of at least m vertices.

The problem of determining $f(n, r, 2)$ have been proposed in [8], and later it was rediscovered in [1]. For $r \leq 5$, $f(n, r, 2)$ have been determined in [2]. The authors of this paper independently proved the following.

THEOREM A [4, 9]. *$f(n, r, 2) \geq n/(r-1)$, and this inequality is sharp if an affine plane of order $r-1$ exists and $r-1$ divides n.*

Applying the linear programming method we genealize Theorem A as follows.

THEOREM 1. *$f(n, r, t) \geq n/q$, where q is the smallest integer satisfying $r \leq q^{t-1} + q^{t-2} + \cdots + q + 1$. The inequality is sharp for $n = q^t m$ and $r = q^{t-1} + q^{t-2} + \cdots + q + 1$ if an $A(t, q)$, the affine space of dimension t and of order q, exists.*

A simple example is $t = 3$, $r = 7$. Theorem 1 says that a 3-cover of an n-set with seven partitions must be of rank at least $n/2$. This was conjectured in [10]. The result is sharp for $n = 8m$.

To see that Theorem 1 is sharp when indicated, consider the t-cover of $A(t, q)$, with r partitions defined by the parallel classes of hyperplanes. Then replace all points of $A(t, q)$ by a set of m points. Since each hyperplane of $A(t, q)$ has q^{t-1} points, the rank of this t-cover is $q^{t-1}m = n/q$.

A further question is that what happens if $r = q^{t-1} + q^{t-2} + \cdots + q + 1$ but n is arbitrary. We do not go into this problem in this paper. It can be treated similarly to the case $t = 2$ in [7].

We give the lower bound for $f(n, r, t)$ in the form

$$f(n, r, t) \geq \frac{n}{\tau^*(r, t)},$$

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where $\tau^*(r, t) = \max\{\tau^*(\mathscr{H})$: $\mathscr{H}$ is an r-partite t-wise intersecting hypergraph$\}$, and $\tau^*(\mathscr{H})$ is the value of an optimal fractional transversal of $\mathscr{H}$. The details will be given in Section 2.

The proof of Theorem 1 is based on the following theorem, which is a special case of a conjecture of Frankl and Füredi ([3], or Conjecture 6.11 in [6]).

THEOREM 2. *Suppose that $\mathscr{H}$ is an r-partite hypergraph such that any two edges intersect in at least s elements. Then $\tau^*(\mathscr{H}) \leq (r-1)/s$.*

The cited conjecture says that 'r-partite' can be replaced by 'r-uniform' in Theorem 2 unless $\mathscr{H}$ is a symmetric (r, s) design. Theorem 2 easily gives the following.

THEOREM 3. *Suppose that $\mathscr{H}$ is an r-partite t-wise intersecting hypergraph and let q be the smallest integer satisfying $r \leq q^{t-1} + q^{t-2} + \cdots + q + 1$. Then $\tau^*(\mathscr{H}) \leq q$.*

In fact, Theorem 3 is essentially the same as Theorem 1, as shown in the next section. Theorems 2 and 3 are proved in Section 3. In Section 4 the case of 'small' r is discussed, and $f(n, r, t)$ is determined for $t < r < 3t/2$.

2. FRACTIONAL TRANSVERSALS AND t-COVERS

A hypergraph $\mathscr{H} = (V, \mathscr{E})$ is a finite set V of *vertices* together with a collection $\mathscr{E}$ of subsets of V, called *edges*. Note that $\mathscr{E}$ may contain the same set more than once. It is convenient to denote the number of edges in $\mathscr{H}$ by $|\mathscr{H}|$. We write $E \in \mathscr{H}$ to indicate that E is an edge of $\mathscr{H}$. For $E \in \mathscr{H}$, $\mathscr{H} - E$ denotes the hypergraph $(V, \mathscr{E} \setminus \{E\})$. The number of edges containing $x \in V$ is the *degree* of x and is denoted by $d(x)$. The maximum of $d(x)$ for $x \in V$ is denoted by $D(\mathscr{H})$. A hypergraph is *r-partite* if its vertex set V can be partitioned into pairwise disjoint sets $V_1, V_2, \ldots, V_r$ such that $|E \cap V_i| = 1$ for each edge $E \in \mathscr{H}$ and $i = 1, 2, \ldots, r$. A set $T \subset V$ is a *transversal* of $\mathscr{H}$ if $T \cap E \neq \emptyset$ for edge $E \in \mathscr{H}$. The minimum cardinality of a transversal of $\mathscr{H}$ is $\tau(\mathscr{H})$, the *transversal number* of $\mathscr{H}$. The dual of $\mathscr{H}$, $\mathscr{H}^*$, is defined as follows: the vertices of $\mathscr{H}^*$ correspond to the edges of $\mathscr{H}$ and the edges of $\mathscr{H}^*$ correspond to the vertices of $\mathscr{H}$, while the vertex-edge incidence is preserved. A hypergraph is *t-wise intersecting* if any t edges have non-empty intersection.

Now we define the main tool in this paper, the fractional transversal number, $\tau^*(\mathscr{H})$, of a hypergraph. A *fractional transversal* of $\mathscr{H} = (V, \mathscr{E})$ is a non-negative function t: $V \to \mathbf{R}^+$ such that $t(E) := \sum_{x \in E} t(x) \geq 1$ for all $E \in \mathscr{H}$. The *value of t* is defined as

$$|t| = \sum_{x \in V} t(x).$$

The *fractional transversal number*, $\tau^*(\mathscr{H})$, is the infimum of $|t|$ over all fractional transversals.

A *fractional matching* of $\mathscr{H} = (V, \mathscr{E})$ is a function w: $\mathscr{E} \to \mathbf{R}^+$ such that

$$w(p) := \sum_{E \ni p} w(E) \leq 1 \qquad \text{for all } p \in V.$$

The value of w is defined as $|w| = \sum_{E \in \mathscr{H}} w(E)$. The *fractional matching number*, $\nu^*(\mathscr{H})$, is the supremum of $|w|$ over all fractional matchings of $\mathscr{H}$.

The duality theorem of linear programming implies that there is an optimal fractional transversal t, and an optimal fractional matching w with $|t| = |w| = \tau^*(\mathscr{H})$. Observe that $w(E) \equiv 1/D(\mathscr{H})$ is always a fractional matching of $\mathscr{H}$. Its value is

$|\mathscr{H}|/D(\mathscr{H})$: therefore $\nu^*(\mathscr{H}) \geq |\mathscr{H}|/D(\mathscr{H})$; that is,

(i) $$D(\mathscr{H}) \geq \frac{|\mathscr{H}|}{\tau^*(\mathscr{H})}.$$

A hypergraph $\mathscr{H}$ is *τ^*-critical* if $\tau^*(\mathscr{H}-E) < \tau^*(\mathscr{H})$ for each edge $E \in \mathscr{H}$.

Let $\mathscr{H}$ be a hypergraph with an optimal fractional matching w. The *support* of w is the set $\{x \in V: w(x)=1\}$. A *maximal* support of $\mathscr{H}$ is a support not contained in any other support of $\mathscr{H}$.

LEMMA 1 [7]. *If $\mathscr{H}$ is τ^*-critical and S is a maximal support, then $|\mathscr{H}| \leq |S|$.*

Consider a t-cover of an n-element set with r partitions. It can be considered as a hypergraph $\mathscr{H}$ with n vertices, the edges of which are the blocks of the partitions. The dual of $\mathscr{H}$, $\mathscr{H}^*$, is an r-partite t-wise intersecting hypergraph with n edges. The rank of the t-cover is $D(\mathscr{H}^*)$. Therefore

(ii) $f(n, r, t) = \min\{D(\mathscr{H})\colon \mathscr{H}$ is r-partite, t-wise intersecting with n edges$\}$.

Introducing

$$\tau^*(r, t) := \max\{\tau^*(\mathscr{H})\colon \mathscr{H} \text{ is } r\text{-partite, } t\text{-wise intersecting}\}$$

(i) and (ii) imply

(iii) $$f(n, r, t) \geq \frac{n}{\tau^*(r, t)}.$$

Therefore a lower bound for $f(n, r, t)$ follows from an upper bound of $\tau^*(r, t)$. Thus, in particular, Theorem 1 follows from Theorem 3. The advantage of (iii) is that an integer extremal value, $f(n, r, t)$, can be estimated by a rational optimum, $\tau^*(r, t)$. The same approach is applied in Section 4.

It is worth mentioning that (iii) can be paralleled by the following upper bound:

(iv) $$f(n, r, t) < \frac{n}{\tau^*(r, t)} + r\tau^*(r, t).$$

To see this, select a τ^*-critical $\mathscr{H}_0$ such that $\tau^*(\mathscr{H}_0) = \tau^*(r, t)$. Define $\mathscr{H}$ from $\mathscr{H}_0$ by taking each edge $E \in \mathscr{H}_0$ with multiplicity $\lceil w(E)n/\tau^*(\mathscr{H}_0)\rceil$, where w is an optimal fractional matching of $\mathscr{H}_0$ with maximal support S. Lemma 1 implies that

$$|\mathscr{H}_0| \leq |S| \leq \sum_{p \in V} \sum_{E \ni p} w(E) = \sum_{E \in \mathscr{H}_0} |E|\, w(E) \leq \tau^*(\mathscr{H}_0) r.$$

Clearly, $|\mathscr{H}| \geq \sum_{E \in \mathscr{H}_0} w(E)n/\tau^*(\mathscr{H}_0)^* n$, and

$$D(\mathscr{H}) < \sum_{E \ni p} \left(\frac{w(E)n}{\tau^*(\mathscr{H}_0)} + 1\right) \leq \frac{n}{\tau^*(\mathscr{H}_0)} + |\mathscr{H}_0| \leq \frac{n}{\tau^*(\mathscr{H}_0)} + r\tau^*(\mathscr{H}_0),$$

proving (iv).

3. PROOFS OF THEOREMS 2 AND 3

PROOF OF THEOREM 2. We may assume that $\mathscr{H}$ is τ^*-critical. Assume indirectly that $\tau^*(\mathscr{H}) > (r-1)/s$,

$$\tau^*(\mathscr{H}) = \frac{r-1}{s} + \alpha \text{ for some } \alpha > 0. \tag{1}$$

Select an optimal fractional matching w with maximal support S. Then $\sum_{E\in\mathcal{H}} w(E) = \tau^*(\mathcal{H})$ and for every edge $E_0 \in \mathcal{H}$ and for every $p \in E_0$ we have

$$w(p) + r - 1 \geq \sum_{x\in E_0} w(x) = \sum_{E\in\mathcal{H}} |E \cap E_0|\, w(E)$$
$$\geq s\tau^*(\mathcal{H}) + (r-s)w(E_0) = r - 1 + s\alpha + (r-s)w(E_0).$$

Thus

$$\frac{w(p) - s\alpha}{r-s} \geq w(E_0) > 0, \tag{2}$$

where $w(E_0) > 0$ follows from $\mathcal{H}$ being τ^*-critical. Now (2) imples that $w(p) - s\alpha > 0$; that is, $\alpha < w(p)/s \leq 1/s$. Therefore (1) yields

$$\tau^*(\mathcal{H}) < r/s. \tag{3}$$

Adding inequality (2) for all edges containing p we obtain

$$d(p) \geq \frac{(r-s)w(p)}{w(p) - s\alpha} > r - s, \tag{4}$$

where the last inequality follows from $\alpha > 0$. Since $(r-1)/s < \tau^*(\mathcal{H}) < r/s$, $\tau^*(\mathcal{H})$ is not an integer; thus

$$\tau^*(\mathcal{H}) > \lfloor \tau^*(\mathcal{H}) \rfloor. \tag{5}$$

Assume that $\mathcal{H}$ has h vertex classes V_i such that $|S \cap V_i| = \lfloor \tau^*(\mathcal{H}) \rfloor$. Applying Lemma 1, we obtain

$$|\mathcal{H}| \leq |S| \leq r(\lfloor \tau^*(\mathcal{H}) \rfloor - 1) + h \qquad (\leq r\lfloor \tau^*(\mathcal{H}) \rfloor). \tag{6}$$

For $h > 0$, let $V_1, V_2, \ldots, V_h$ be the vertex classes with $|S \cap V_i| = \lfloor \tau^*(\mathcal{H}) \rfloor$. Since V_i is a transversal of $\mathcal{H}$, and $|V_i| \geq \tau^*(\mathcal{H}) > \lfloor \tau^*(\mathcal{H}) \rfloor$, we can choose $v_i \in V_i \setminus S$ for $i = 1, 2, \ldots, h$. Let $T := \{v_1, \ldots, v_h\}$.

If an edge $E \in \mathcal{H}$ contains $v_i \in T$, then

$$\tau^*(\mathcal{H}) = \sum_{x\in V_i} w(x) = \sum_{x\in S\cap V_i} w(x) + \sum_{x\notin S\cap V_i} w(x)$$
$$= \lfloor \tau^*(\mathcal{H}) \rfloor + \sum_{x\notin S\cap V_i} w(x) \geq \lfloor \tau^*(\mathcal{H}) \rfloor + w(v_i).$$

Therefore $w(v_i) \leq \tau^*(\mathcal{H}) - \lfloor \tau^*(\mathcal{H}) \rfloor = \{\tau^*(\mathcal{H})\}$, where $\{\ \}$ denotes the fractional part. Applying this to (2) we obtain

$$w(E) \leq \frac{w(v) - s\alpha}{r-s} \leq \frac{\{\tau^*(\mathcal{H})\} - s\alpha}{r-s} < \frac{\{\tau^*(\mathcal{H})\}}{r-s} \tag{7}$$

for $v \in E \cap T$, $E \in \mathcal{H}$.

Let $\mathcal{H}' = (V', \mathcal{E}')$ be the hypergraph with $V' = V$, $\mathcal{E}' = \{E \in \mathcal{H}: E \cap T \neq \emptyset\}$.

Claim. *For $h > 0$, $|\mathcal{H}'| > h$.*

Proof. First we show that there is no edge $E^* \in \mathcal{H}'$ with $|E^* \cap T| > r - s$. Suppose the contrary. Then $|E \cap E^*| \geq s$ implies $E \cap T \neq \emptyset$ for all $E \in \mathcal{H}$; that is, $\mathcal{H}' = \mathcal{H}$. Thus (7) implies

$$\tau^*(\mathcal{H}) = \sum_{E\in\mathcal{H}} w(E) = \sum_{E\in\mathcal{H}'} w(E) < |\mathcal{H}'| \frac{\{\tau^*(\mathcal{H})\}}{r-s} = |\mathcal{H}| \frac{\{\tau^*(\mathcal{H})\}}{r-s}.$$

Applying (6) and the inequality $\lfloor y\rfloor\{y\}\leqslant y-1$, which holds for all $y\geqslant 1$, we continue the previous inequality as follows:

$$\tau^*(\mathcal{H})<|\mathcal{H}|\frac{\{\tau^*(\mathcal{H})\}}{r-s}\leqslant\frac{r\lfloor\tau^*(\mathcal{H})\rfloor\{\tau^*(\mathcal{H})\}}{r-s}\leqslant\frac{r(\tau^*(\mathcal{H})-1)}{r-s}.$$

We conclude that $\tau^*(\mathcal{H})\geqslant r/s$, contradicting (3).

Thus $|E^*\cap T|\leqslant r-s$ holds for all $E^*\in\mathcal{H}'$. Then (4) implies

$$|\mathcal{H}'|\geqslant\frac{\sum_{x\in T}d(x)}{r-s}>\frac{|T|\,(r-s)}{r-s}=h,$$

proving the claim. ■

Returning to the proof of Theorem 2, we have $w(E)<1/(r-s)$ by (2), and $w(E')<\{\tau^*(\mathcal{H})\}/(r-s)$ for all $E'\in\mathcal{H}'$ by (7). Hence $\tau^*(\mathcal{H})$ can be estimated as follows:

$$\begin{aligned}\tau^*(\mathcal{H})&=\sum_{E\in\mathcal{H}\setminus\mathcal{H}'}w(E)+\sum_{E'\in\mathcal{H}'}w(E')\leqslant\frac{|\mathcal{H}|-|\mathcal{H}'|}{r-s}+\frac{\{\tau^*(\mathcal{H})\}}{r-s}|\mathcal{H}'|\\&=\frac{|\mathcal{H}|-|\mathcal{H}'|\,(1-\{\tau^*(\mathcal{H})\})}{r-s}.\end{aligned}$$

But $|\mathcal{H}|\leqslant r(\lfloor\tau^*(\mathcal{H})\rfloor-1)+h$ by (6), and $|\mathcal{H}'|>h$ by the Claim, so we have

$$\begin{aligned}|\mathcal{H}|-|\mathcal{H}'|\,(1-\{\tau^*(\mathcal{H})\})&<r(\lfloor\tau^*(\mathcal{H})\rfloor-1)+h-h(1-\{\tau^*(\mathcal{H})\})\\&=r(\lfloor\tau^*(\mathcal{H})\rfloor-1)+h\{\tau^*(\mathcal{H})\}\leqslant r(\lfloor\tau^*(\mathcal{H})\rfloor-1)+r\{\tau^*(\mathcal{H})\}=r\tau^*(\mathcal{H})-r.\end{aligned}$$

Therefore $\tau^*(\mathcal{H})<(r\tau^*(\mathcal{H})-r)/(r-s)$ giving $r/s<\tau^*(\mathcal{H})$, contradicting (3).

This implies $\alpha\leqslant 0$, and $\tau^*(\mathcal{H})\leqslant(r-1)/s$ follows. □

PROOF OF THEOREM 3. Use the notation $q^{\langle i\rangle}=q^i+q^{i-1}+\cdots+q+1$, $q^{\langle 0\rangle}=1$. If $|E\cap F|\geqslant q^{\langle t-2\rangle}$ for all $E,F\in\mathcal{H}$, then one can apply Theorem 2 with $s=q^{\langle t-2\rangle}$.

$$\tau^*(\mathcal{H})\leqslant\frac{r-1}{q^{\langle t-2\rangle}}\leqslant\frac{q^{\langle t-1\rangle}-1}{q^{\langle t-2\rangle}}=q.$$

So, we may suppose that there exist $E_1^2, E_2^2\in\mathcal{H}$ with

$$|E_1^2\cap E_2^2|\leqslant q^{\langle t-2\rangle}-1. \tag{8}$$

Let a be the largest integer such that there exist a edges $E_1^a, E_2^a,\ldots,E_a^a\in\mathcal{H}$ with

$$\left|\bigcap_{i=1}^{a}E_i^a\right|\leqslant q^{\langle t-a\rangle}-1. \tag{9}$$

Here $2\leqslant a$ by (8), and $a\leqslant t-1$ since $\mathcal{H}$ is t-wise intersecting. Set $Z=\bigcap_{1\leqslant i\leqslant a}E_i^a$. The definition of a implies that

$$|Z\cap E|\geqslant q^{\langle t-a-1\rangle} \tag{10}$$

holds for all $E\in\mathcal{H}$.

Define the following fractional transversal $t\colon V\to\mathbf{R}^+$ of $\mathcal{H}$

$$t(x)=\begin{cases}1/q^{\langle t-a-1\rangle} & \text{for } x\in Z,\\ 0 & \text{otherwise.}\end{cases}$$

In equality (10) shows that t is really a fractional transversal of $\mathcal{H}$, and (9) implies that $\tau^*(\mathcal{H})\leqslant|t|\leqslant(q^{\langle t-a\rangle}-1)/q^{\langle t-a-1\rangle}=q$. □

4. t-covers With Few Partitions

It is easy to prove that $f(n, t, t) = n$ ([9]) and follows also from Theorem 1. In other words, a t-cover with t partitions must include the whole underlying set as a block. Equivalently, if the edges of a complete t-uniform hypergraph are colored by t colors, then some color class determines a connected subhypergraph. The case $t = 2$ was observed by Erdős and Rado.

If $r > t$ but r is close to t (say, $r < 2^t$), then the lower bound of Theorem 1 is $n/2$. Better estimates can be given; in fact, $f(n, r, t)$ can be determined for $t < r < 3t/2$.

Theorem 4. *Suppose that $t < r < 3t/2$, and let $\mathscr{H}$ be an r-partite, t-wise intersecting hypergraph. Then*

$$\tau^*(\mathscr{H}) \leqslant 1 + \frac{2}{3t - r + 1}.$$

Proof. It was proved in [F2] that the conclusion of Theorem 4 holds for every *r-uniform* t-wise intersecting hypergraphs $\mathscr{H}$, unless $\mathscr{H}$ contains one of six special substructures. It is easy to check that these substructures cannot occur in an *r-partite* hypergraph; thus Theorem 4 is a corollary of Theorem 3.8 from [5]. □

Using (iii) from Section 2, Theorem 4 implies the lower bound for $f(n, r, t)$ in the following theorem.

Theorem 5. *Suppose that $t < r < 3t/2$, and let γ represent $(3t - r + 1)/(3t - r + 3)$. Let $n \equiv l \bmod (3t - r + 3)$, $0 \leqslant l < 3r - t + 3$. Then*:
(a) $f(n, r, t) = \lceil \gamma n \rceil$, *for* $l < (3t - r + 3)/2$ *or* $l \geqslant t + 1$; *and*
(b) $\lceil \gamma n \rceil \leqslant f(n, r, t) \leqslant \lceil \gamma n \rceil + 1$, *otherwise* (*i.e. for* $(3t - r + 3)/2 \leqslant l \leqslant t$).

Proof (construction). Let $n = k(3t - r + 3) + l$ for some integer $k \geqslant 1$. First, consider the case $l = 0$. Partition the n-element set V into sets $A_i (1 \leqslant i \leqslant 3(r - t + 3))$ and B_j $(1 \leqslant j \leqslant 3t - 2r - 3)$, where

$$|A_i| = k \qquad \text{and} \qquad |B_j| = 2k. \tag{11}$$

Now the r partitions of a t-cover will be defined as follows. Every partition has two blocks, so it is enough to define only one block, P_i, for each partition $i = 1, 2, \ldots, r$. The first $3(r - 1 - t)$ blocks form triangle-like structures, for $1 \leqslant i \leqslant r - 1 - t$ set

$$P_{3i-2} = A_{3i-2} \cup A_{3i-1}, \qquad P_{3i-1} = A_{3i-2} \cup A_{3i}, \quad ; \quad P_{3i} = A_{3i-1} \cup A_{3i}.$$

The rest of the blocks are the B_i's: for $1 \leqslant i \leqslant 3t - 2r - 3$ let

$$P_{i+3r-3-3t} = B_i.$$

It is easy to see that this is a t-cover of rank $k(3t - r + 1)$. This rank is equal to $\lceil \gamma n \rceil$, the lower bound in Theorem 5.

If $n = k(3t - r + 3) + l$, where $0 \leqslant l \leqslant t$, then distribute l extra vertices arbitrarily among the sets A_i's and B_j's, but at most one extra vertex goes to one set. Then, the rank of the obtained t-cover is $k(3t - r + 1) + l$. In the case $l < (3t - r + 3)/2$, this rank equals $\lceil \gamma n \rceil$; otherwise it is $\lceil \gamma n \rceil + 1$.

If $l \geqslant t + 1$, then modify the definition (11) in the following way:

$$|A_i| = \begin{cases} k + 1 & \text{if } i \equiv 0 \text{ or } 1 \mod 3, \\ k \text{ or } k + 1' & \text{otherwise,} \end{cases}$$

$$|B_j| = k + 1 \qquad \text{for all } j$$

The rank of the obtained t-cover is $k(3t - r + 1) + l - 1$, and this equals $\lceil \gamma n \rceil$. □

For $r = t + 1$ and $t + 2$ case (a) holds in Theorem 5. For $r = t + 1$, we obtain $f(n, r, r-1) = \lceil n(r-1)/r \rceil$ $(r \geq 4)$, as was proved in [9].

With more work one can improve the lower bound to show that in case (b) the upper bound is the true value of $f(n, r, t)$.

Acknowledgement

This research was supported in part by the Hungarian National Science Foundation under Grant No. 1812.

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Received 9 February 1990 and Accepted 19 July 1991

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