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RESEARCH PROBLEMS

Problem 114. Posed by Akira Saito.

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A graph G is *k-extendable* if G has a perfect matching and for any set E of k independent edges in G, there is a perfect matching of G that uses the edges of E. Characterize the connected graphs G which are k-extendable but such that $G + xy$ is not k-extendable for any non-adjacent pair of vertices x and y.

For $k = 1$, G is either $K_{r,r}$ or K_{2r} for $r \in \{1, 2, 3, \ldots\}$.

Problem 115. Posed by András Gyárfás.

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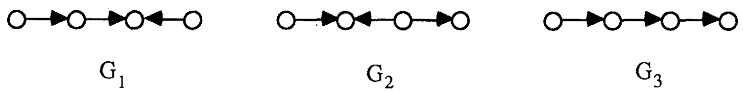
Consider the family $\boldsymbol{G}_i$, $i = 1, 2, 3$, of graphs having an acylic orientation without containing an induced subdigraph isomorphic to the digraph G_i below. Let $\mathrm{cl}(G)$ denote the clique number of a graph G, that is, the maximum order of a complete subgraph of G, and let $\mathrm{chr}(G)$ denote the chromatic number of G. We are interested in the relation of $\mathrm{cl}(G)$ and $\mathrm{chr}(G)$ for graphs in $\boldsymbol{G}_i$, $i = 1, 2, 3$.

V. Chvátal [1] proved that $\mathrm{chr}(G) = \mathrm{cl}(G)$ for all $G \in \boldsymbol{G}_1$. On the other hand, for all n, there exists a graph $H_n \in \boldsymbol{G}_2$ such that $\mathrm{cl}(H_n) = 2$ and $\mathrm{chr}(H_n) = n$.

What can be said about the graphs in $\boldsymbol{G}_3$? Is it true that the chromatic number



of these graphs is less than a function of the clique number? In particular, is it true that $\mathrm{cl}(G) = 2$ implies $\mathrm{chr}(G) \leqslant c$ for all $G \in \boldsymbol{G}_3$?

References

[1] V. Chvátal, Perfectly ordered graph, in: Topics on Perfect Graphs, Ann. Discrete Math. 21 (1984) 63–65.