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Ramsey and Turán-type problems in bipartite geometric graphs

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We study geometric versions of Ramsey-type and Turán-type problems for bipartite graphs. We consider balanced bipartite geometric graphs $G(n,n) = [A\,;B]$ in the following natural representation. The vertex classes A, B of $G(n,n)$ are represented in $\mathbb{R}^2$ as

$$A = \{(1,0),(2,0),\ldots,(n,0)\}, \quad B = \{((1,1),(2,1),\ldots,(n,1)\}$$

and the edge ab is the line segment joining $a \in A$ and $b \in B$ in $\mathbb{R}^2$. This model is essentially the same as the cyclic bipartite graphs and ordered bipartite graphs considered earlier by several authors. Subgraphs — paths, trees, double stars, matchings — are called non-crossing if they do not contain edges with common interior point.

We determine the maximum number of edges in a bipartite geometric graph $G(n,n)$ that does not contain

a. non-crossing matchings with $k+1$ edges;

b. matchings with $k+1$ pairwise crossing edges;

c. non-crossing trees with $k+1$ vertices — in this case we also show that any graph with more edges than the extremal value contains a non-crossing double star with $k+1$ vertices.

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We determine the Ramsey number of non-crossing double stars: in every 2-coloring of a geometric $K_{n,n}$ there is a non-crossing monochromatic double star with at least $4n/5$ vertices and this is best possible. To find the Turán number of non-crossing paths and the Ramsey number of non-crossing subtrees and paths remain open problems.