

Uniform Approximate Functional Equation for Principal L-Functions (Erratum)

Gergely Harcos

In [1], the auxiliary function $F(s, \pi_\infty)$ was defined by (3.2) as the square root of a certain quotient of exponential and gamma functions. While the quotient is holomorphic in the half plane $\Re s > -1/(m^2 + 1)$, it was erroneously concluded and used in the paper that the same holds for $F(s, \pi_\infty)$. I am indebted to Florin Spinu for bringing this problem to my attention. In this note, I correct the error, so that all the theorems and corollaries of the paper hold true in their original form.

The above-mentioned difficulty can be resolved by defining $F(s, \pi_\infty)$ slightly differently as

$$F(s, \pi_\infty) = \frac{1}{2} C^{-s/2} N^s \frac{L\left(\frac{1}{2} + s, \pi_\infty\right) L\left(\frac{1}{2}, \tilde{\pi}_\infty\right)}{L\left(\frac{1}{2} - s, \tilde{\pi}_\infty\right) L\left(\frac{1}{2}, \pi_\infty\right)} + \frac{1}{2} C^{s/2}. \quad (1)$$

Here, C is the analytic conductor of π at the central point as given by [1, (2.4)]. This new auxiliary function is holomorphic in $\Re s > -1/(m^2 + 1)$ and has the same features as the original choice to make the argument work out properly: it is of moderate growth in vertical strips, satisfies the functional equation (3.3) and the symmetry (3.5) in [1], and $F(0, \pi_\infty) = 1$.

In fact, only a few small adjustments need to be made in the rest of the paper. First, the new notation turns (3.14) into

$$2C^{-s/2}F(s, \pi_\infty) - 1 \ll_\sigma (1 + |s|)^{md\sigma}, \quad \Re s = \sigma. \quad (2)$$

Correspondingly, (3.22) should read

$$\pi^{mds} \frac{L\left(\frac{1}{2} + s, \pi_\infty\right)}{L\left(\frac{1}{2} - s, \tilde{\pi}_\infty\right)} \ll_{\sigma, m, d} \left(\frac{\pi^{md}C}{N}\right)^\sigma |s|^{md\sigma}, \quad \Re s = \sigma. \quad (3)$$

Second, all the four occurrences of $C^{-it/2}F(it, \pi_\infty)$ on page 931 should be replaced by $2C^{-it/2}F(it, \pi_\infty) - 1$. Correspondingly, (4.8) becomes

$$i\Re \sum_{j=1}^{md} \left\{ \frac{\Gamma'}{\Gamma} \left(\frac{1}{4} + \frac{\mu_j}{2} + \frac{it}{2} \right) - \log \left(\frac{1}{4} + \frac{\mu_j}{2} \right) \right\}. \quad (4)$$

Finally, I take the opportunity to record a misprint in [1]. The formulae in (2.3) should read

$$L(s, \pi_\infty) = \prod_{j=1}^{md} \pi^{-(s+\mu_j)/2} \Gamma\left(\frac{s+\mu_j}{2}\right), \quad L(s, \tilde{\pi}_\infty) = \prod_{j=1}^{md} \pi^{-(s+\bar{\mu}_j)/2} \Gamma\left(\frac{s+\bar{\mu}_j}{2}\right). \quad (5)$$

References

- [1] G. Harcos, *Uniform approximate functional equation for principal L-functions*, Int. Math. Res. Not. **2002** (2002), no. 18, 923–932.

Gergely Harcos: Mathematics Department, The University of Texas at Austin, 1 University Station C1200, Austin, TX 78712-0257, USA

E-mail address: gharcos@math.utexas.edu