Some problems related to my lectures (The Erdős-Szekeres theorem and its relatives) at the First Mexican Winter School in Discrete Mathematics, Guanajuato, 2010. I will try to add some more.

1. Let $f(n)$ be smallest number with the property that among $f(n)$ points in the plane in general position there are always n in convex position. The best known bounds are

$$2^{n-2} + 1 \leq f(n) \leq \binom{2n-5}{n-2} + 1.$$

a. (probably hard) "Essentially" there is only one lower bound example, found by Erdős and Szekeres. Try to find a different one!

b. Improve the upper bound!

c. (extremely hard, or hopeless) find the value of $f(n)$.

2. For any $d \geq 2$, let $f_d(n)$ be smallest number with the property that among $f(n)$ points in the d dimensional space in general position (no $d+1$ on a $d-1$-dimensional hyperplane) there are always n in convex position. The best known bounds are

$$2^{c_1 \sqrt[d-1]{n}} \leq f(n) \leq 2^{c_2 n}.$$

Find a better lower bound!

3. Is it true for some $\varepsilon > 0$ that n points in the plane in general position always determines at least $(1+\varepsilon)n^2 - o(n^2)$ empty triangles?

4. Is there a constant K with the following property? For any $n > 0$ there exists a set of n points in the plane in general position such that on every pair of points (edge) there are at most K empty triangles?

5. Let $g_5(n)$ (resp. $g_6(n)$) be the minimum number of empty convex pentagons (resp. hexagons) deternined by n points in the plane in general position. The best known bounds are

$$3\left\lfloor \frac{n-4}{8} \right\rfloor \leq g_5(n) \leq 1.02n^2,$$

$$\left\lfloor \frac{n-5}{1712} \right\rfloor \leq g_6(n) \leq 0.02n^2.$$

a. Improve these bounds! The lower bounds seem much easier to improve.

b. Find the order of magnitude of $g_5(n)$ and $g_6(n)$.

6. Is it true that for n sufficiently large, n points in the plane in general position, colored with two colors, always determine a convex, monochromatic, empty quadrilateral?

7. Let $\Delta(n)$ be the minimum number of empty monochromatic triangles determined by n points in the plane in general position, colored with two colors. The best known bounds for $\Delta(n)$ are

$$c_1 n^{4/3} \leq \Delta(n) \leq c_2 n^2.$$

a. Improve these bounds! The lower bounds seem much easier to improve. (Of course I might be wrong!)

b. Find the order of magnitude of $\Delta(n)$. (Conjecture: n^2)

8. This problem came into my mind during preparation for the lectures, it might be known, trivial, very hard, I don't know. Find a set of n ponts (actually a sequence of sets) where the number of empty quadrilaterals grows asymptotically faster than the number of empty triangles.

9. It is known now, that any set of 2760 points in the plane in general position, colored with two colors, determines a monochromatic, empty (but not necessarily convex) quadrilateral.

a. Of course, 2760 could not be the best bound. Improve it!

b. The present proof is quite complicated, find a simpler proof, even if it gives a weaker bound.

10. It is known that among $F_4(n) \leq c_1 n^3$ disjoint convex sets, every four of which are in convex position, there are n in convex position. The best known lower bound for $F_4(n)$ is quadratic. Find the order of magnitude of $F_4(n)$. (Conjecture: quadratic.)

11. It is known that among $S_4(n) \leq 2^{2^{cn}}$ segments (not necessarily disjoint), every four of which are in convex position, there are n in convex position. The best known lower bound for $S_4(n)$ is again quadratic. Find the order of magnitude of $F_4(n)$, or at least improve the upper bound.