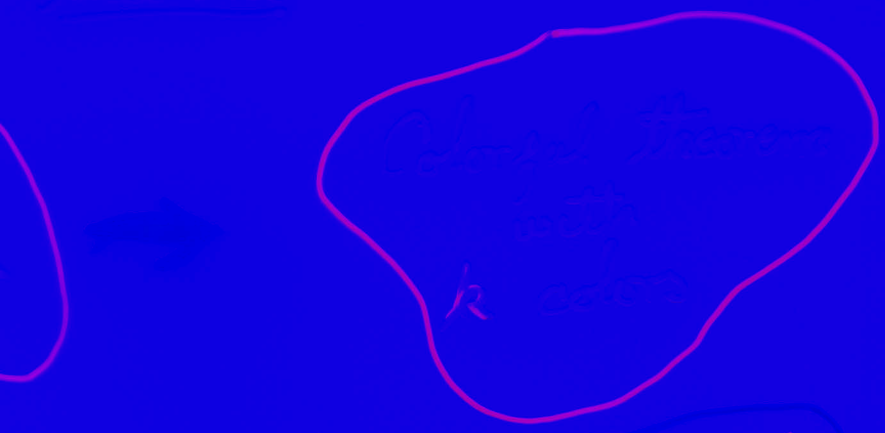
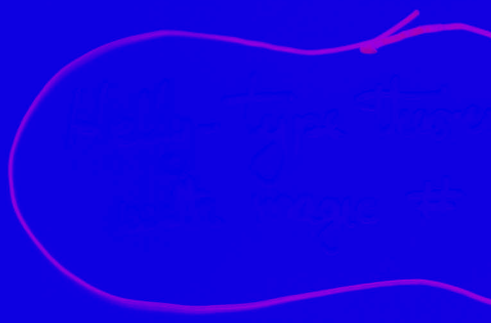


- Set $\mathcal{H}$ painted with k - colors
- Hypothesis

"Every subset of $\mathcal{H}$ of size k heterochromatic has property P

there is a color (let say red) such that the subset of $\mathcal{H}$ consisting of red objects has property P



- Property P
- Set $\mathcal{H}$
- Magic # k

Colorful Results are

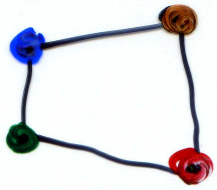
Example: Red, Green, blue points in the space

"Every heterochromatic triple lie on a line"

Conclusion: All Points lie on a line (3-colors)

Example: Red, Green, blue orange points in the pla

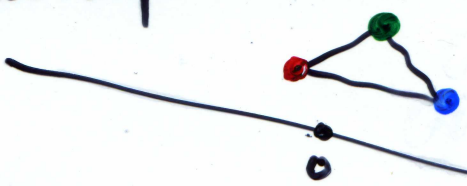
"Every heterochromatic set of 4 points is on convex pos



"No conclusion"

Example: Caratheodory Red, Green blue points in the plane

"Every heterochromatic triple lie on a halfplane through the or

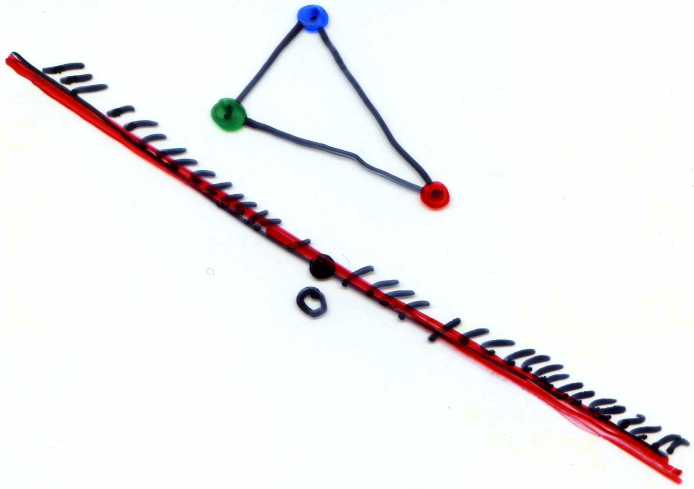


Conclusion: there are two colors such that all points of these two colors lie on a halfplane through the origin.

Colorful Caratheodory (Bárány) in the plane

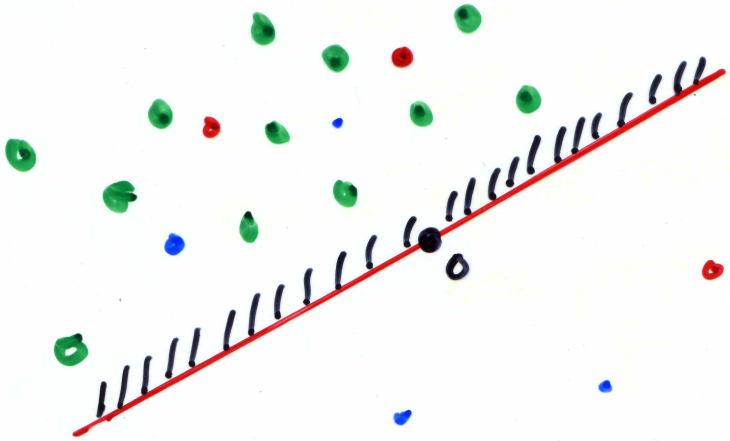
Set → Red, Green and blue points in $\mathbb{R}^2 - \{0\}$

Hypothesis → "Every heterochromatic triangle



lies in a halfplane through the origen

Conclusion → there is a color (let say green) such that all green points lie in a halfplane through the origen



Colorful Krasnolewsky's theorem M.

Exposition of

Picasso

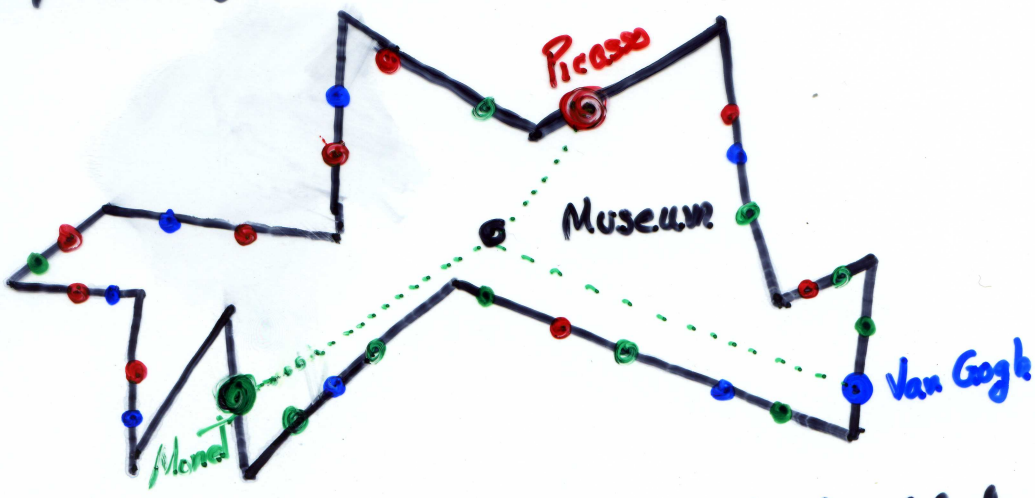
Monet

and

Van Gohg

"paintings"

"Suppose given a Picasso painting, a monet painting and a Van Gogh painting there is a point in the Museum from which these three paintings can be seen



Then, from one of these three artists (let say Picasso) there is a point in the Museum from which all Picasso paintings can be seen.

Usually the conclusion is
"for one color"

sometimes the conclusion does not exist.

and sometime the conclusion is for
several colors

theorem: the Colorful Caratheodory theorem in $\mathbb{R}^d$ is true for the conclusion two colors.

Arocha, Bárány, Bracho, Fabila, M. 2008
independently

Pach, Holmsen, Tverberg 2008 (general version

Tverberg Theorem is a special case of Colorful Kirchberger theorem for k-types

Intervals on $\mathbb{R}$

Intervals on $\mathbb{R}^2$

Family of Re

Family of Blue

ϕ

Red $\cap$ blue $\neq$

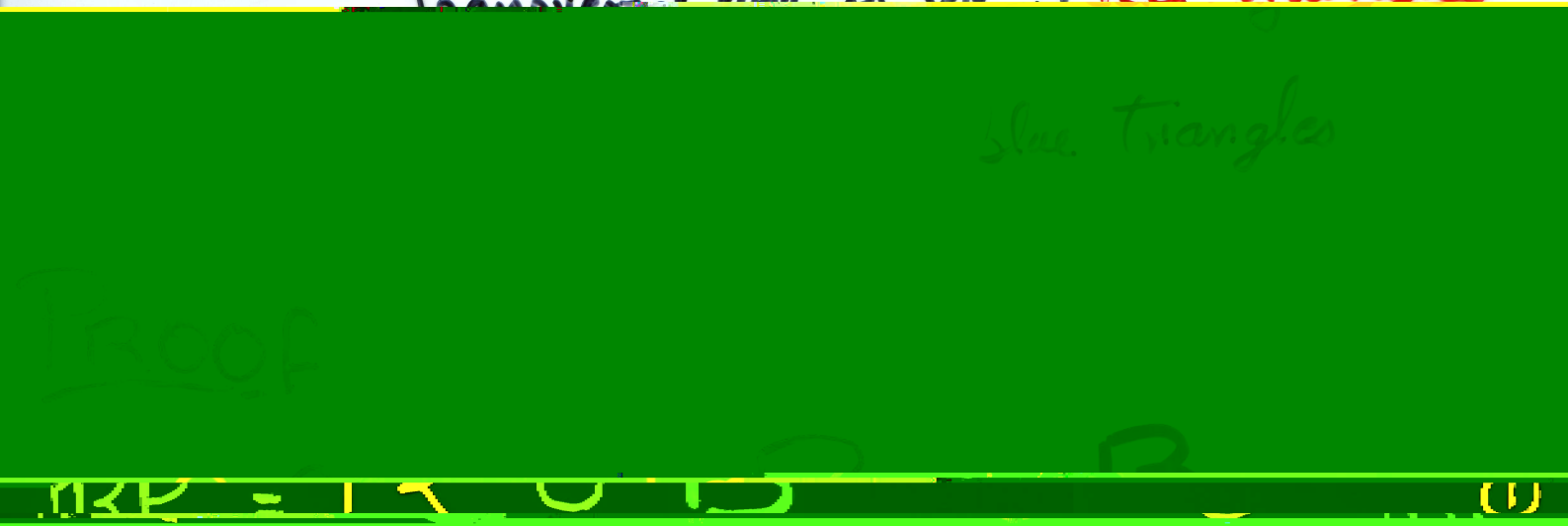
or

-3-

The theory of Topological or Virtual Transversals to a family of convex sets

topological or Virtual Transversals

Theorem: In every embedding of K in $\mathbb{R}^3$ either



A direction d is blue if orthogonal to d there is a plane transversal to the 3 blue triangles

Every direction is either blue or red

Given a line l through the origen
Proyect orthogonally K

"We have 3 red intervals and 3 blue intervals."

(red interval) $\cap$ (blue interval) $\neq \emptyset$

Either the 3 red intervals are intersecting or the 3 blue intervals are intersecting

orthogonal to l

By Lusternik - Schnirelmann

$\mathbb{RP}^2$ R U B

R

B

Red triangles

blue triangles

theorem. 3 red, 3 blue, 3 green $\subset \mathbb{R}^4$

$(\text{red}) \cap (\text{blue}) \cap (\text{green}) \neq \emptyset$

$\Downarrow$

there is a color and a line transversal to all convex sets of this color

---

theorem 4 red, 4 blue, 4 green $\subset \mathbb{R}^4$

$\text{red} \cap \text{blue} \cap \text{green} \neq \emptyset$

$\Downarrow$

either there is a color and a line transversal to all convex sets of this color

or there is a hyperplane transversal to the 12-convex sets

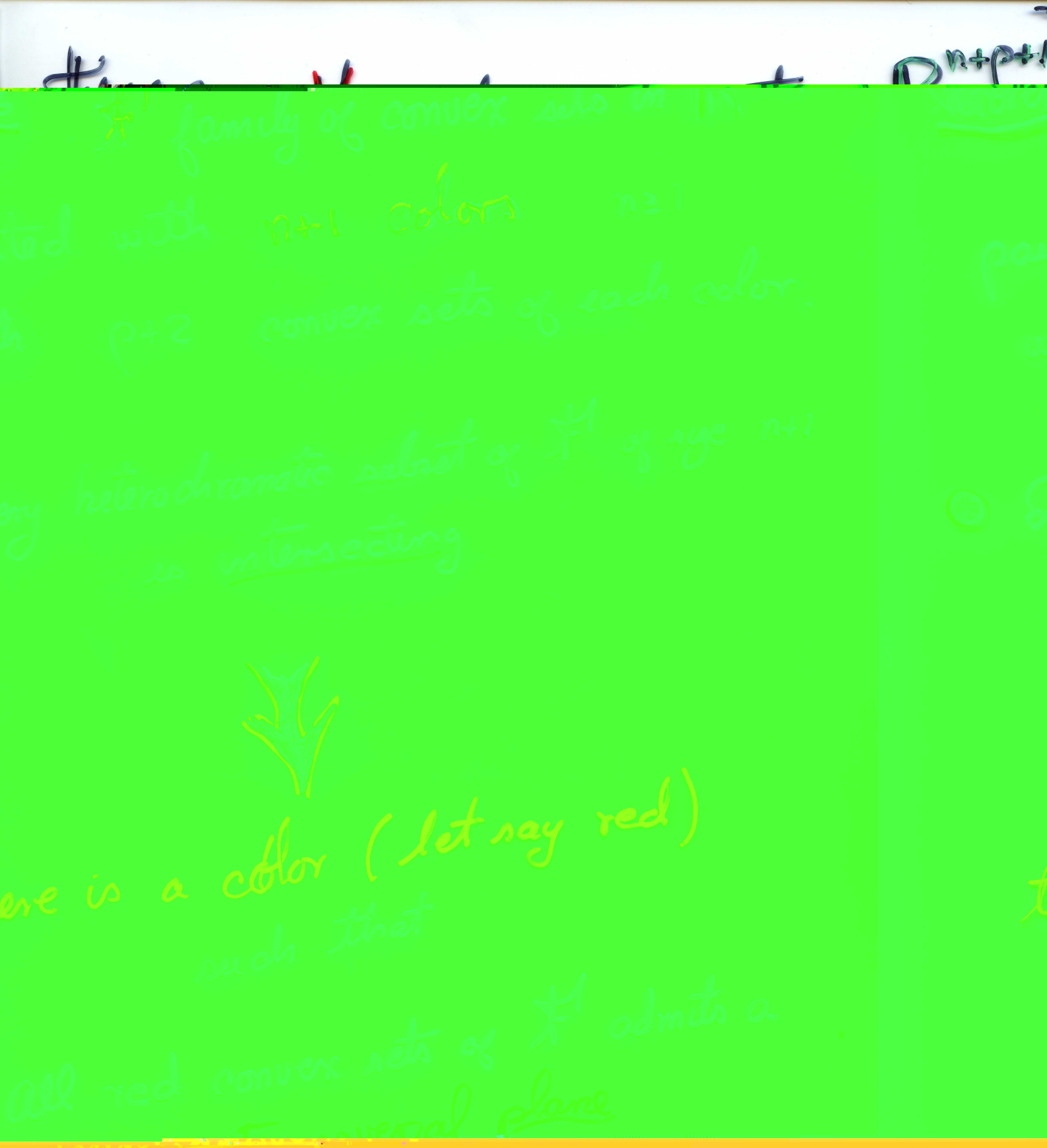
---

theorem 4 red 4 blue, 4 green $\subset \mathbb{R}^4$

every time we have a red, a blue and a green convex set one of them intersect the other tw

$\Downarrow$

there is a color and a plane transversal to all convex sets of this color


ere is a color (let say red)

Structural topological theorems

$A_0, A_1, A_2 \ldots A_{p+1}$

convex sets in $\mathbb{R}^d$.

$a_0 \in A_0, \quad a_1 \in A_1 \ldots \quad a_{p+1} \in A_{p+1}$

without p-transversal

$\pi(a_0, \ldots) \cong T'(A_0, \ldots A_{p+1})$

Affine Gelfand MacPherson

Configurations of $r+1$ points in $\mathbb{R}^m$

$$G_r^m \cong G(r, m)$$

$\downarrow$ homotopy equivalence.