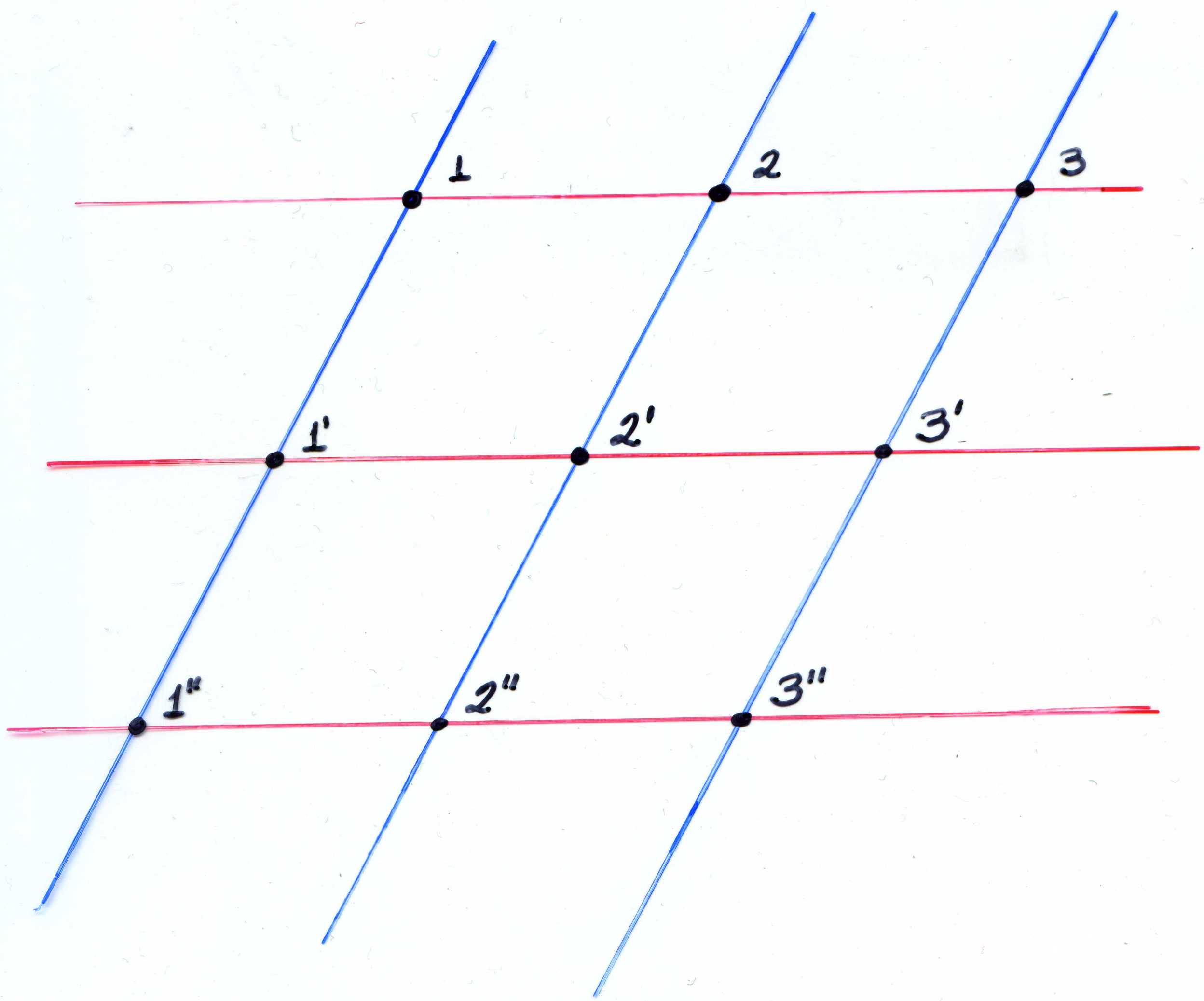
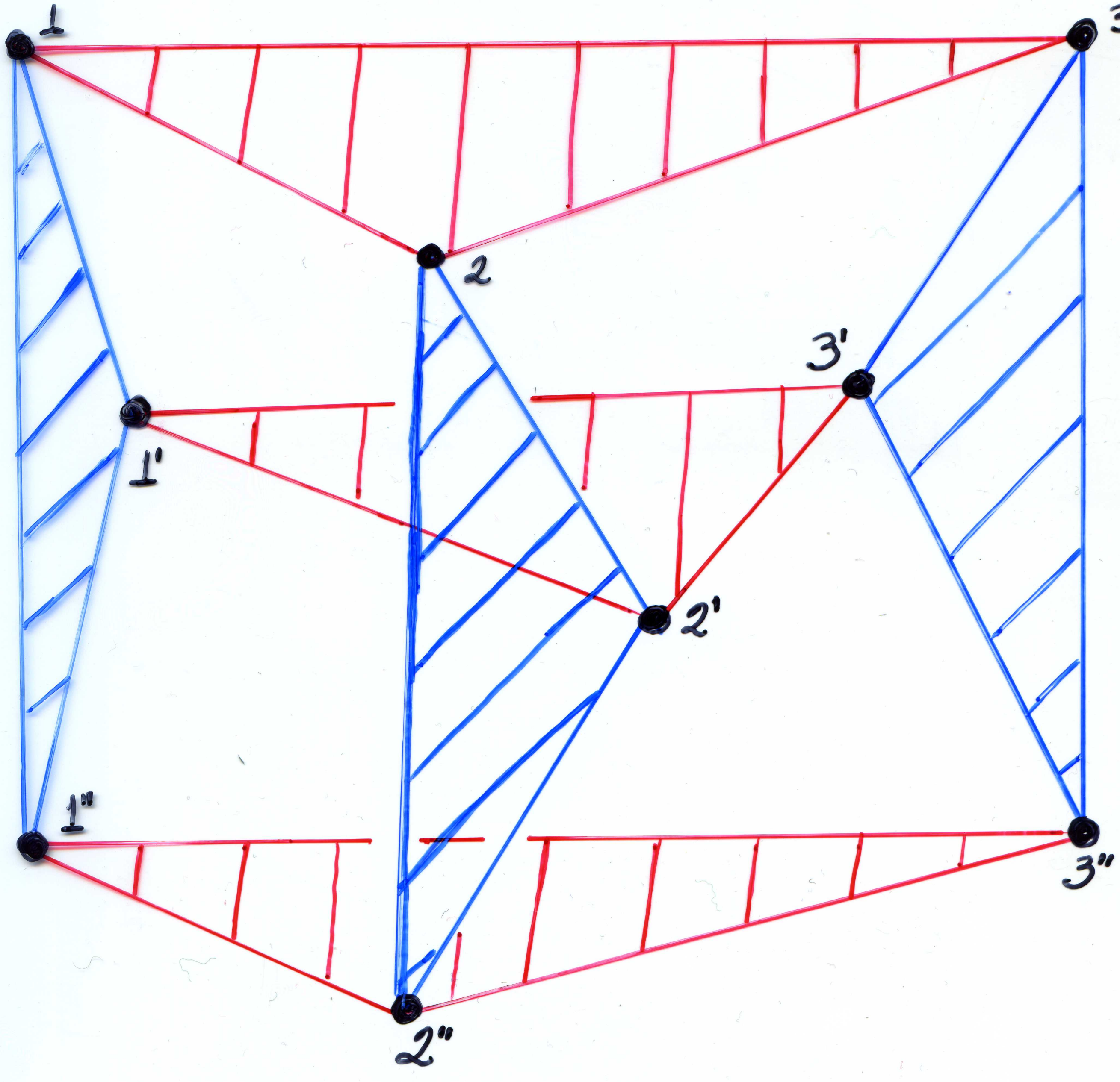
Let us
Consider the following
configuration of
points and lines



Let K be its

corresponding

3- Hypergraph
or
Simplicial complex



For every linear embedding

$K \subset \mathbb{R}^3$

there is:

Either a <u>line</u> <u>transversal</u> to the

three red triangles

or

a <u>line</u> <u>transversal</u> to the

three blue triangles

there is an embedding of

K in $\mathbb{R}^4$

without {
line transversals to the 3 $\triangle$'s
line transversals to the 3 $\triangle$'s

but if this is so

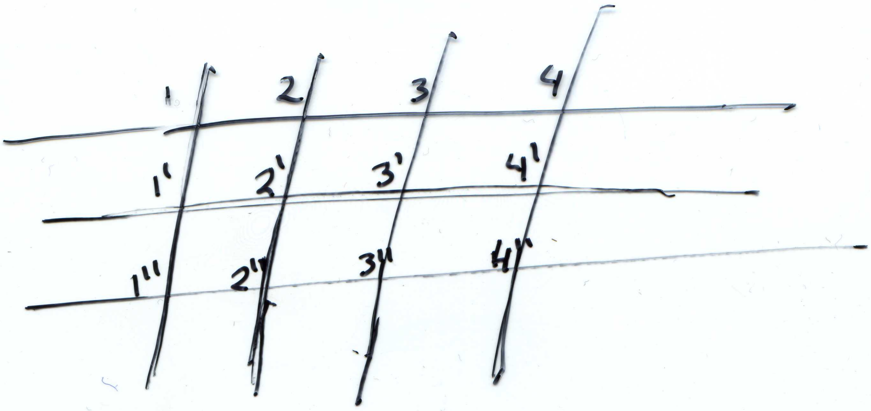
there is a plane transversal to

any 4 of the 6

Triangles

of

K



3 horizontal l
4 vertical l

Simplicial complex K
with

12 points
3 tetrahedra (horizontal lin
4 triangles (vertical lines)

<u>Claim</u> For Every linear embedding
$K \subset \mathbb{R}^4$

either
there is a line through the 3 tetrahedr
or
there is a plane through the 4 triangles

Ingredients

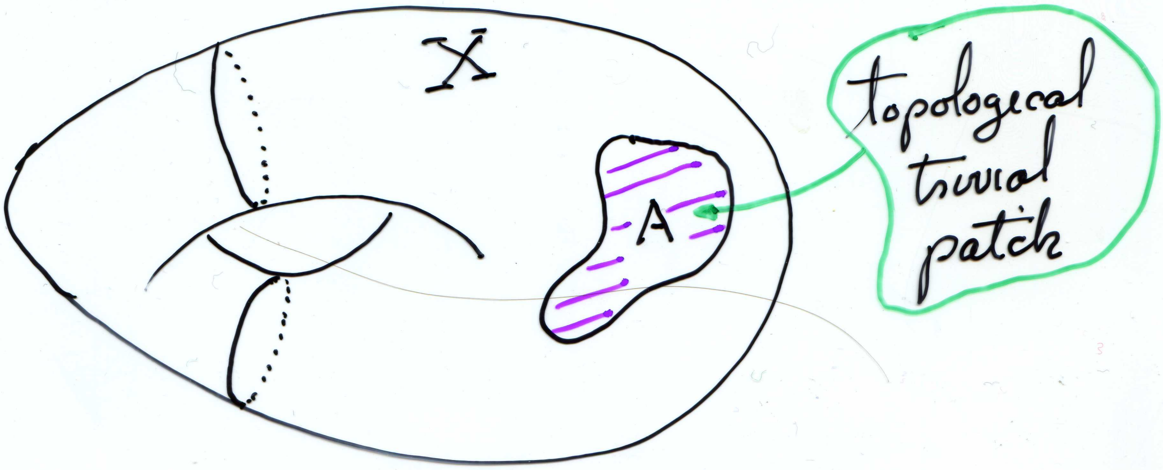
1.- the Lusternik- Schnirelmann category (topology

2.- the Colorful Helly-type theorems in the spirit of Lovász and Bárány (Combinatorial Geometry

3. the theory of topological or virtual transversals to a family of convex sets (topology)

1. The Category of Lusternik-Schnirelmann

- Global Calculus of Variations
- Existence of closed geodesics on surfaces



Definition $A^{\text{closed}} \subset X$ is a trivial patch if A does not contain an essential curve of X

Main Question. With how many trivial patches it is possible to cover X ?

Impossible to cover Proyective 2-Space $\mathbb{R}P^2$ with only two trivial patches

$$\mathbb{R}P^2 = A \cup B$$

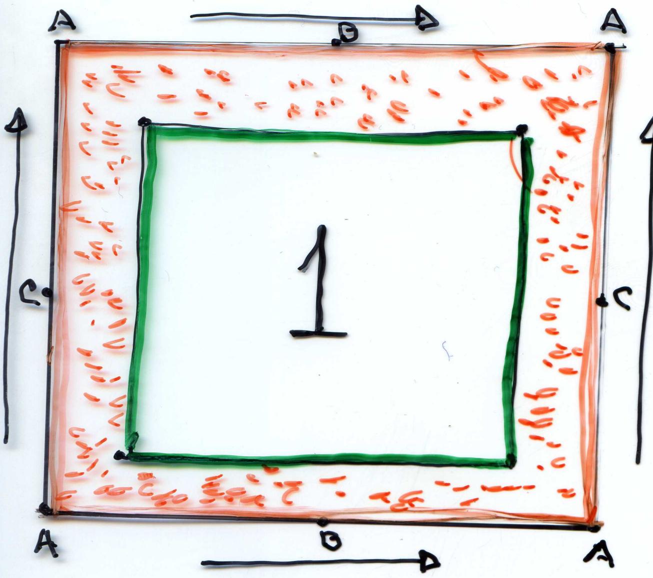
$$A^{\text{closed}} \subset \mathbb{R}P^2$$

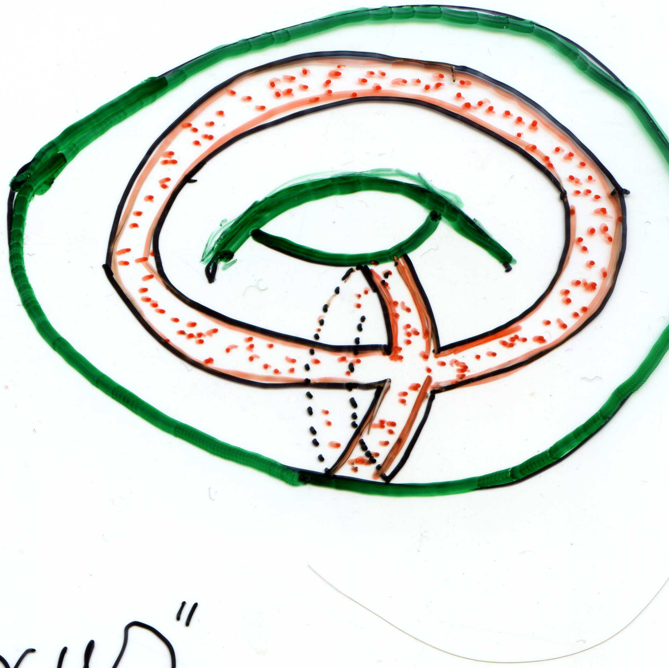
$$B^{\text{closed}} \subset \mathbb{R}P^2$$

Either A contains an essential curve of $\mathbb{R}$

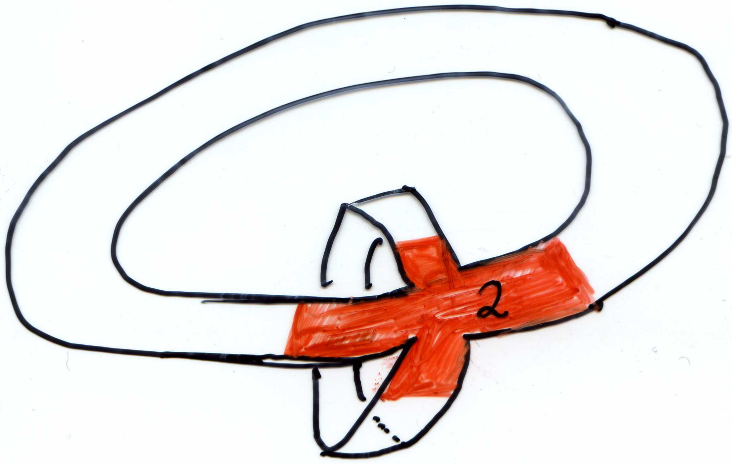
or

B contains an essential curve of $\mathbb{R}P^2$.

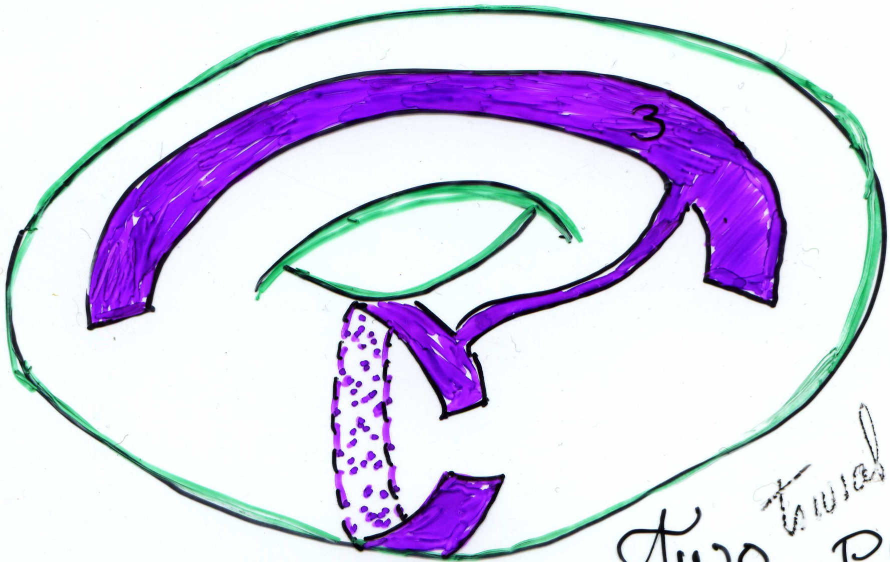




"Torus"



3 trivial patches



Imposs
with
two trivial patches

Impossible to cover Projective n Space

$\mathbb{R}P^n$

with only n trivial patches

Borsuk-Ulam

g more topology
it is possible to have
uivalent information for

Grassmannian Manifolds
$G(n, d)$

le for the space
of lines in 3-Space.

Usi

equ

For exam

-2-

the Colorful Helly-type theorems in the spirit of Lovasz and Barany

Helly's theorem in the Plane.

Set — a finite family of convex sets in the plane

Property — has non-empty intersection

$\Longleftrightarrow$

every subfamily of size

Magic number — 3

has nonempty intersection

Helly-Type Theorems.

Set — a set F

Property — has property P

$\Longleftrightarrow$

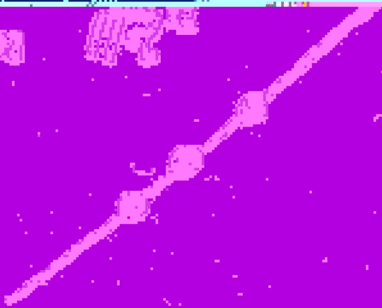
every subfamily of F of size

Magic Number — k

has property P

Example 1

... of points on the line



Set → finite collection ...

Property → to be in ...

Shape of → 3

Example 2

... points on the plane ... position

Set → finite collection of p...

Property → to be in convex ...

Shape of → 4

Example 3

Example 7 Kirchenberger's Theorem

Set → Finite collection of points in $\mathbb{R}^d$ of 2 types $\{\circ, \square\}$

Property → there is a halfplane that separates

d points from square points

Magic # →

Bracho, Bárány, Fabila, M.

llection of points in $\mathbb{R}^d$ of k-types

Example 8

Set →

Property

Magic # → $(k-1)(d+1)+1$

Colorful theorems

Colorful Helly theorem (Lovász) in the plane

Sets painted with three colors

Red, Green, blue.

Convex

Suppo

every (a red convex set, a green convex set and a blue convex set)

is intersecting

$\Downarrow$