

HOMEWORK 5

1. Show that if G is a k -critical graph (i.e. $\chi(G) = k$ but $\chi(G - e) = k - 1$ for every edge $e \in E(G)$) then for every vertex $x \in V(G)$ there is a k -coloring of G such that x is the only one vertex of color k .

2*. Construct a graph G_k for every $k \geq 4$ such that for every vertex $x \in V(G)$ there is a k -coloring of G_k such that x is the only one vertex of color k but G is NOT k -critical. (Notice that there is no such a graph for $k = 3$.)

3*. Show that the graphs obtained by the Mycielski construction are k -critical.