A Ramsey-type Theorem (in Hungarian)

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For pairwise disjoint finite sets $H_1, H_2, \cdots, H_m$, let $[H_1, \cdots, H_m]^t$ denote the family of all sequences $(A_1, \cdots, A_m)$ with $A_j \subset H_j$, $|A_j| = t$. The author proves the following theorem: For any natural numbers $m, t, k, i_1, \cdots, i_k$ there exists an $n = n_0(m, t, k, i_1, \cdots, i_k)$ such that whenever $[H_1, \cdots, H_m]^t$ is partitioned into k classes and $|H_j| \geq n$ for $j = 1, \cdots, m$, then there exist $Q_1, \cdots, Q_m$ with $Q_j \subset H_j$, $|Q_j| = i_j$ such that $[Q_1, \cdots, Q_m]^t$ lies entirely in one of the classes. This generalizes Ramsey's well-known theorem (the case $m = 1$). The proof uses multiple induction on m, t and k.

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Corollary *Let k be an integer, let R be an N-dimensional brick. Then there exists an integer $n = n(k, R)$ such that for any k-coloration of the n-dimensional Euclidean space there exists a monochromatic copy (the vertices have the same color) of R.*