HOMEWORK SET #12 / CO1A / Spring 2009

1. Find a pair of orthogonal Latin squares of order 9.
2. Are there two essentially different Latin squares of order 3, that is such that none of them can be obtained from the other by permutation of the rows, columns and numbers?
3. Are the following two Latin squares essentially different, that is can you get one of them from the other by permutation of the rows, columns and numbers?

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix}$$

4. If there exists a pair of orthogonal Latin squares of order n and A is a Latin square of order n, A is not necessarily a member of an orthogonal pair of Latin squares. Give an example to illustrate this. (Hint: questions no 3 and 4 are related to each other).

5.) In a Steiner triple system with $v = 9$, find b and r.

6.) The following nine blocks form a part of a Steiner triple system with nine elements:

$$\{a,b,c\},\ \{d,e,f\},\ \{g,h,i\},\ \{a,d,g\},\ \{c,e,h\},\ \{b,f,i\},\ \{a,e,i\},\ \{c,f,g\},\ \{b,d,h\},$$

How many missing blocks are there? Add additional blocks that will lead to a Steiner triple system.

7.) If a town has 924 clubs, each club has 21 members and any 2 persons belong to exactly 2 clubs jointly, then how many inhabitants does the town have? How many clubs does each person belong to? (Don't be surprised: this is a very small town, and everybody belongs to many clubs)

8.) The following 10 blocks are from a BIBD on 8 elements with $\lambda = 3$: $\{a,b,c,d\}$, $\{b,c,f,g\}$, $\{a,d,e,h\}$, $\{a,b,e,f\}$, $\{c,d,g,h\}$, $\{e,f,g,h\}$, $\{a,d,f,g\}$, $\{b,c,e,h\}$, $\{a,b,g,h\}$, $\{c,d,e,f\}$ How many further blocks are there and which are they?

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