HOMEWORK SET #8 / CO1A / Spring 2009

1. Determine all graphs with exactly one pair of vertices of equal degree (all other degrees are distinct).
2. Prove that a graph G contains a circuit of length at least $k+1$ if $d(x) \geq k$ for all $x \in V(G)$.
3. Show that the complement of a disconnected graph is connected!
4. Give an example of a graph on 8 vertices which is isomorphic to the complement of itself.
5. Give an example of a graph G with $\chi(G) = 2$ and a coloring of G with the greedy algorithm (that is an ordering of the vertices of G in which order the greedy algorithm colors the vertices of the graph) requiring n colors (for every n big enough)
6. For every $n \geq 3$ give an example of a graph G having $\chi(G) \geq n$ but $G \not\supseteq K_n$.
7. What can be the chromatic number of a connected simple graph with exactly two cycles?
8. Prove that for every graph G on the vertex set V there is a partition of $V = V_1 \cup V_2$ such that if $G(V_1)$ and $G(V_2)$ denote the graphs spanned by G on the sets V_1 and V_2 respectively, than $\chi(G(V_1)) + \chi(G(V_2)) = \chi(G)$.

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