HOMEWORK SET #5 / CO1A / Spring 2009

1.) Prove that $\sum_{k=1}^{n} \frac{(-1)^{k-1}}{k}\binom{n}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$.
2.) Suppose that $G(x) = \frac{x}{x^2-3x+2}$ is the ordinary generating function of the sequence (a_k). Find a_k.
3.) Solve the recurrence relation under the given initial conditions: $a_n = 2a_{n-1} + 4a_{n-2} - 8a_{n-3}$ $a_0 = 0,\ a_1 = 0,\ a_2 = 8$.
4.) Solve the recurrence relation under the given initial conditions: $a_k = 6a_{k-1} - 11a_{k-2} + 6a_{k-3}$, $a_0 = 1,\ a_1 = 4,\ a_2 = 9$.
5.) We have n forints. Every day we buy exactly one of the following products: pretzel (1 forint), candy (2 forints), icecream (2 forints). What is the number of possible ways of spending all the money (the order of the bought products counts)?
6.) Solve the recurrence relation under the given initial conditions: $c_n = 9c_{n-1} - 15c_{n-2} + 7c_{n-3}$, $c_0 = 0,\ c_1 = 1,\ c_2 = 2$.
7.) Solve the recurrence relation under the given initial conditions: $a_{n+2} = 2a_n - a_{n+1} + 3 \cdot (-2)^n$, $a_0 = -1$, $a_1 = 1$.
8.) Solve the recurrence relation under the given initial conditions: $k_n = k_{n-1} + n + 6$, $k_0 = 0$.

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