

HOMEWORK SET #3 / CO1A / Spring 2009

1. What is the coefficient of x^{100} in the product $(1 + x + x^2 + x^3 + x^4 + \dots) \times (1 - x + x^2 - x^3 + x^4 - \dots) \times (1 - x^2 + x^4 - x^6 + x^8 - \dots)$?
2. Find the ordinary generating function of the sequence n^2 , i.e. find a close formula for the function $\sum_{n=0}^{\infty} n^2 \cdot x^n$.
3. Find the sequence (a_k) such that the function $\frac{1}{1+x^4}$ is the ordinary generating function of (a_k) , that is $\frac{1}{1+x^4} = \sum_{n=0}^{\infty} a_n \cdot x^n$.
4. Find the sequence (b_k) such that the function $\frac{b}{a+x}$ is the ordinary generating function of (b_k) , that is $\frac{b}{a+x} = \sum_{n=0}^{\infty} b_n \cdot x^n$.
5. In each of the following, denote by a_n (with an appropriate choice of n) the answer to the question, set up the appropriate generating function for the sequences a_n , indicate what coefficient (which value of n) you are looking for, and finally calculate the answer to the question.
 - a. In how many different ways can $4n$ letters be selected from $2n$ A's, $2n$ B's and $2n$ C's without order?
 - b. Find the number of solutions to the equation $x_1 + x_2 + x_3 + x_4 = 18$ where for each x_i $1 \leq x_i \leq 7$ (the order of the variables counts).
 - c. How many ways can we place 25 indistinguishable balls into 8 distinguishable cells with no empty cell?
 - d. In how many ways can a total of 80 be obtained if 50 distinguishable dice are rolled?
6. What is the coefficient of x^{10} in the product $(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots) \times (1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots) \times (x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots)$?
7. A codeword over the alphabet $\{0, 1, 2, 3\}$ consists of at least one of each of the digits 0,1,2 and 3, and has length 20. How many such codewords are there?
8. A codeword from the alphabet $\{a, b, c, d, e, f\}$ is legitimate iff a and c appear even and b and d appear odd number of times (no constrain on e and f). Find the number of legitimate codewords of length k .

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