HOMEWORK SET # 2 second version / CO1A / Spring 2009

1. Let $B(n)$ denote the number of partitions of an n element set into *any* number of parts, i.e., $B(n) = \sum_{i=1}^{n} S(n,i)$. Prove that

$$B(n+1) = \sum_{i=0}^{n} \binom{n}{i} B(i).$$

2. A man has 15 different pairs of shoes. How many ways can he choose a right shoe and a left shoe that do not match?
3. Show (preferably by a combinatorial argument) that $\sum_{k=1}^{n} k\binom{n}{k}2^{k-1}2^{n-k} = n(4^{n-1})$.
4. Compute $\mathrm{S}(n, n-5)$.
5. Find the number of ways to partition a set of 25 elements into exactly 5 subsets (I mean here a concrete positive integer).
6. A value function on a set A assigns 0 or 1 to each subset of A. How many different value functions are there on a set A of n elements?
7. Show (in two different ways if possible) that

$$\binom{n}{0} + \binom{n+1}{1} + \cdots + \binom{n+r}{r} = \binom{n+r+1}{r}.$$

8. Show that

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}.$$

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