

HOMEWORK SET #1 / CO1A/ Spring 2009

1. How many odd numbers between 1000 and 9999 have at least one even digit?
2. In how many different ways can we choose 12 cans of soup if there are 5 different varieties available, if
  - a.) the order of the chosen cans doesn't count.
  - b.) the order of the chosen cans does count.
3. Prove that  $\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$ .
4. Prove that for every  $n \geq 2$   $\sum_{k=2}^n k(k-1)\binom{n}{k} = n(n-1)2^{n-2}$ .
5. The 5 players of the Chicago Bull are all of distinct heights. In how many different ways can they enter the court if no player is placed between two others both higher than him? Generalize for  $n$  players!
6. In how many ways can we form three triplets of nine police officers for patrols (the order of the patrols and order of the officers in the patrols do not count).
7. How many numbers less than 1 million contain the digit 2?
8. Prove that  $\sum_{k=0}^n \frac{1}{k+1} \binom{n}{k} = \frac{1}{n+1} (2^{n+1} - 1)$ .

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