HOMEWORK SET #3 / CO1A / Fall 2009

1. What is the coefficient of x^{100} in the product $(1+x+x^2+x^3+x^4+\cdots)\times(1-x+x^2-x^3+x^4-\cdots)\times(1-x^2+x^4-x^6+x^8-\cdots)$?
2. Find the ordinary generating function of the sequence n^2, i.e. find a close formula for the function $\sum_{n=0}^{\infty} n^2\cdot x^n$.
3. Find the sequence (a_k) such that the function $\frac{1}{1+x^4}$ is the ordinary generating function of (a_k), that is $\frac{1}{1+x^4}=\sum_{n=0}^{\infty} a_n\cdot x^n$.
4. Find the sequence (b_k) such that the function $\frac{b}{a+x}$ is the ordinary generating function of (b_k), that is $\frac{b}{a+x}=\sum_{n=0}^{\infty} b_n\cdot x^n$.
5. In each of the following, denote by a_n (with an appropriate choice of n) the answer to the question, set up the appropriate generating function for the sequences a_n, indicate what coefficient (which value of n) you are looking for, and finally calculate the answer to the question.
   a. In how many different ways can 4n letters be selected from $2n$ A's, $2n$ B's and $2n$ C's without order?
   b. Find the number of solutions to the equation $x_1+x_2+x_3+x_4=18$ where for each x_i $1\le x_i\le 7$ (the order of the variables counts).
   c. How many ways can we place 25 indistinguishable balls into 8 distinguishable cells with no empty cell?
   d. In how many ways can a total of 80 be obtained if 50 distinguishable dice are rolled?
6. What is the coefficient of x^{10} in the product $(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots)\times(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots)\times(x+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots)$?
7. A codeword over the alphabet $\{0,1,2,3\}$ consists of at least one of each of the digits 0,1,2 and 3, and has length 20. How many such codewords are there?
8. A codeword from the alphabet $\{a,b,c,d,e,f\}$ is legitimate iff a and c appear even and b and d appear odd number of times (no constrain on e and f). Find the number of legitimate codewords of length k.

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