HOMEWORK SET #2 / CO1A / Fall 2009

1. Prove that the number of partitions of a number n into no more than r terms is equal to the number of partitions of n into any number of terms, each at most r. (A partition of a number n into r terms is a collection of positive integers $\{a_1, a_2, a_3, \ldots, a_r\}$ such that $a_1+a_2+\cdots+a_r = n$; note that order does not count!)
2. Show (preferably by a combinatorial argument) that $S(n, k) = kS(n-1, k) + S(n-1, k-1)$.
3. Show by a combinatorial argument that
$$S(n+1, k) = \binom{n}{0}S(0, k-1) + \binom{n}{1}S(1, k-1) + \cdots + \binom{n}{n}S(n, k-1).$$
4. Compute $S(n, 4)$, S$(n, n-3)$ and S$(n, n-4)$.
5. In how many ways can we partition the number n into exactly k parts, if the the order of the parts counts?
6. In how many different ways can you reach the bottom-right corner of a chessboard from the top-left corner, if you may only step right and downward and you may never be above the top-left bottom-right diagonal of the board.
7. In how many ways can 12 binary digits be picked (with order) if each must be picked an even number of times (that is what is the number of codewords of length 12 over $\{0, 1\}$ with both digits occurring even number of times)?
8. In how many ways can 3n letters be selected from $3n$ A's, $3n$ B's and $3n$ C's (without order)?

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