

Asymptotic behavior of flows i

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The flow and the network

$G = (V, E)$ graph, s space parameter, t time parameter

$$\frac{\partial}{\partial t} u_j(s, t) = c_j(s) \frac{\partial}{\partial s} u_j(s, t) + q_j(s) \cdot u_j(s, t)$$

$$u_j(s, 0) = f_j(s), \quad s \in (0, 1),$$

$$\phi_{ij}^- u_j(1, t) = \omega_{ij} \sum_{k=1}^m \phi_{ik}^+ u_k(0, t), \quad t \geq 0,$$

Locally:

$$e_i \quad \omega_{il} \quad \bullet \quad v_i$$

Globally: strongly connected directed graph

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The semigroup

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$$A = \begin{pmatrix} c_1(s) \frac{\partial}{\partial s} & & 0 \\ & \ddots & \\ 0 & & c_m(s) \frac{\partial}{\partial s} \end{pmatrix} + \begin{pmatrix} M_{q_1} & & \\ & \ddots & \\ 0 & & M_{q_m} \end{pmatrix}$$

$$u'(t) = Au(t)$$

The semigroup

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State spaces:

$$\mathfrak{L} := ((L^1[0, 1])^m, \|\cdot\|_{\mathfrak{L}})$$

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strongly continuous



w^* -continuous

positive irreducible semigroup *positive irreducible* semigroup

What we get for 'free'

A real flow...

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'translation' semigroup on the edges

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'translation' semigroups on the edges

'computable' local behavior

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edge-vertex duality

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Positivity & Irreducibility & Compact resolvent

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Positivity & Irreducibility & Compact resolvent

spectral bound = growth bound contained

$$\omega_0 = s(A) \in \sigma(A) \text{ (on } \mathfrak{L} \text{)}$$

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unique positive constant flow

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unique positive constant flow

spectral decomposition

The spectrum

$\lambda \in \sigma(A) \iff 1 \in \sigma(\mathbb{A}_\lambda)$ where

$$(\mathbb{A}_\lambda)_{ip} = \begin{cases} \omega_{pj} \exp(\xi_j(0,1) - \lambda \tau_j(0,1)), \\ 0, \end{cases}$$

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$$\xi_j(0,1) = \int_0^1 \frac{q_j(s)}{c_j(s)} ds \text{ mass gain/loss on } e_j$$

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Asymptotic behavior of

$A \rightsquigarrow (T(t)_{t \geq 0})$ strongly continuous, positive, ir

$$\tilde{q} = \sup \{ \operatorname{Re} \lambda : \lambda \in \sigma(A) \}$$

$$\tilde{A} := A - \tilde{q}I \rightsquigarrow \tilde{T}(t)$$

$\dim \ker \tilde{A} = 1$ with positive eigenvector

$$\mathfrak{L} = \mathfrak{L}_1 \oplus \mathfrak{L}_2$$

$$\mathfrak{L}_1 = \overline{\operatorname{lin}} \left\{ x \in D(\tilde{A}) \mid \exists \alpha \in \mathbb{R} : \tilde{A}x = \mathbf{i}\alpha x \right\} \quad ?$$

$$\tilde{T}(t)x \rightarrow 0 \iff x \in \mathfrak{L}_2$$

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Only the average speeds r

$$\tau_j(0, 1) = \int_{s_1}^{s_2} \frac{ds}{c_j(s)} \text{ travel time on } e_j$$

$\{\tau_{j_1}(0, 1) + \dots + \tau_{j_k}(0, 1) : e_{j_1}, \dots, e_{j_k} \text{ form a cycle}\}$

dependent

linearly

over \mathbb{Q}

independent

If $(LD_{\mathbb{Q}})$:

$$c(\tau_{j_1}(0, 1) + \dots + \tau_{j_k}(0, 1)) \in \mathbb{N}$$

$$l(c) = \gcd \{c(\tau_{j_1}(0, 1) + \dots + \tau_{j_k}(0, 1)); e_{j_1}, \dots, e_{j_k}\}$$

$$\gamma = \frac{l(c)}{c}$$

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Complete description of asy

	\mathcal{L}
$(LD_{\mathbb{Q}})$	$\Gamma \cdot e^{t\sigma(\tilde{A})} = \Gamma \cdot \sigma(\tilde{T}(t)) \setminus \{0\} \quad (t \geq 0)$ $S(t) = \tilde{T}(t) _{\mathcal{L}_1}, \ \tilde{T}(t) - S(t)\ _{\mathcal{L}} \leq Me^{-\varepsilon t}$ $\sigma(\tilde{A}) \cap i\mathbb{R} = 2\pi i \frac{1}{\gamma} \mathbb{Z}$
$(LI_{\mathbb{Q}})$	$\mathcal{L}_0 = \mathcal{L}_1$ $\sigma(\tilde{A}) \cap i\mathbb{R} = \{0\}$ $\eta \in \sigma(\tilde{A})$ $R_{\varepsilon, K} = \{z \in \mathbb{C} : \operatorname{Re} \eta - \varepsilon \leq \operatorname{Re} z \leq \operatorname{Re} \eta + \varepsilon,$ $\qquad \qquad \qquad \downarrow \qquad \qquad \qquad \operatorname{Im} \eta + K \leq \operatorname{Im} z\}$ $\sigma(\tilde{A}) \cap R_{\varepsilon, K} \neq \emptyset$ $\tilde{T}(t) \rightarrow Pr_{\mathcal{L}_0} \text{ strongly}$

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$(LD_{\mathbb{Q}})$	$\Gamma \cdot e^{t\sigma(\tilde{A})} = \Gamma \cdot \sigma(\tilde{T}(t)) \setminus \{0\} \ (t \geq 0)$ $S(t) = \tilde{T}(t) _{\mathfrak{L}_1}, \ \tilde{T}(t) - S(t)\ _{\mathfrak{L}} \leq Me^{-\varepsilon t}$ $\sigma(\tilde{A}) \cap i\mathbb{R} = 2\pi i \frac{1}{\gamma} \mathbb{Z}$	
$(LI_{\mathbb{Q}})$	$\mathfrak{L}_0 = \mathfrak{L}_1$ $\sigma(\tilde{A}) \cap i\mathbb{R} = \{0\}$ $\eta \in \sigma(\tilde{A})$ $R_{\varepsilon, K} = \{z \in \mathbb{C} : \operatorname{Re} \eta - \varepsilon \leq \operatorname{Re} z \leq \operatorname{Re} \eta + \varepsilon,$ $\quad \downarrow \quad \quad \quad \operatorname{Im} \eta + K \leq \operatorname{Im} z\}$ $\sigma(\tilde{A}) \cap R_{\varepsilon, K} \neq \emptyset$ $\tilde{T}(t) \rightarrow Pr_{\mathfrak{L}_0}$ strongly	

$$\forall \varepsilon > 0 \forall \varphi \in C[0, 1]^m \exists t > 0 : \forall \mu \in \mathfrak{M} \forall \tau > t \left(\left| \langle \tilde{T}(t) \mu, \varphi \rangle \right| > \varepsilon \right)$$

What had to be left out

negative coupling

speed of convergence to periodic orbit

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negative coupling

speed of convergence to periodic orbit

orbits on large periodic structures

flows/diffusions on fractals

END

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speed of convergence to periodic orbit

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