Corrigendum to
Secret sharing on infinite graphs

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Abstract

The proof of Claim 6.8 in the Appendix of [1] is incorrect. Here we give a new (and hopefully correct) proof.

Key words. Secret sharing scheme, information theory, infinite graph, lattice.

1 Introduction

The proof of Claim 6.8 in the Appendix of [1] is incorrect. I am indebted to Prof. Hamiredza Maimani [2] who called my attention to the error.

2 The new proof

Claim 2.1 *The information ratio of the graph G depicted on figure 1 is 2.*

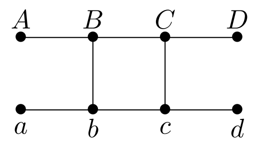

Figure 1: The graph G

Proof The proof of the first part of the claim, namely that $R(G) \le 2$ was correct. G is a spanned subgraph of the 2-lattice, and the 2-lattice has information ratio 2. For proving the lower bound we use the method outlined in the paper [1]. Let f be any function satisfying the Shannon inequalities (a)–(e) enlisted there, we claim that

$$f(bc) + f(BC) \ge 8. \tag{1}$$

As $f(b) + b(c) + f(B) + f(C) \ge f(bc) + f(BC) \ge 8$, at least one of $f(b)$, $f(c)$, $f(B)$, and $f(C)$ must be ≥ 2, thus the lower bound 2 follows.

To get inequality (1) we use instances of the Shannon inequalities (a)–(e) as follows:

$$\begin{aligned}
f(a) + f(b) &\ge f(ab) \\
f(ab) + f(bc) &\ge 1 + f(b) + f(abc) \\
f(acBD) - f(acD) &\ge f(acABD) - f(acAD) \ge 1 \\
f(acBCD) - f(acBD) &\ge 1 \\
f(ac) - f(a) &\ge f(acC) - f(aC) \\
f(acC) - f(aC) &\ge 1 + f(acBCD) - f(aBCD) \\
f(abc) - f(ac) &\ge f(abcD) - f(acD) \\
\hline
f(bc) &\ge 4 + f(abcD) - f(aBCD).
\end{aligned}$$

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Now the graph G is invariant under the following permutation of the vertices: $a \leftrightarrow D$, $b \leftrightarrow C$, $c \leftrightarrow D$, $d \leftrightarrow A$, thus applying this transformation to the above inequality we get another valid inequality for our graph:

$$f(CB) \geq 4 + f(DCBa) - f(Dcba).$$

Adding these latter two inequalities we get (1), as required. $\square$

References

[1] L. Csirmaz: Secret sharing on infinite graphs, Tatra Mt. Math. Publ **41** (2008) pp 1–18

[2] Hamidreza Maimani: Personal communication, 2009 November