MATH 309

Practice problems for the midterm exam

1. Prove that there are 5 platonic solids.

2. How many faces and vertices does the dodecahedron have? What are its faces?

3. Prove that the area of the fundamental parallelogram of a planar lattice is independent of the basis.

4. Let T be a transformation of the plane that satisfies

$$T((1,2)) = (2,3)\ ,\ T((1,3) = (-1,2))\ ,\ T((2,5)) = (1,5).$$

Is it possible that T is a linear transformation?

5. Let T be a transformation of the plane that satisfies

$$T((1,2)) = (2,2)\ ,\ T((2,5)) = (3,5)\ ,\ T((1,3)) = (2,3).$$

Is it possible that T is a linear transformation?

6. Write up the matrix of the linear transformation f for which

$$f((1,2)) = (5,1)\ ,\ f((-1,2)) = (3,3)\,.$$

7. Let f be the rotation by $\pi/3$ and g be the reflection to the line through the origin of angle $\pi/2$. What are the matrices of f and g? What are the matrices of $f \circ g$ and $g \circ f$? What are these product transformations?

8. Prove by matrix multiplication that the product of a reflection and a rotation is a reflection.

9. List all the symmetries of a regular hexagon.

10. Let Λ be the lattice generated by the vectors $(1,2)$ and $(2,1)$. Choose a basis consisting of vectors of length at least 4. Also, choose a basis consisting of vectors whose angle is at least $2\pi/3$.