Lefschetz fibration structures on knot surgery 4-manifolds

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Abstract

Since it was known that any closed symplectic 4-manifold admits a Lefschetz pencil and a Lefschetz fibration structure can be obtained from a Lefschetz pencil by blowing-up the base loci, a study on Lefschetz fibrations have become an important research theme to understand symplectic 4-manifolds topologically.

By the way, R. Fintushel and R. Stern introduced a new surgery technique, called *a knot surgery*, to show that a large class of simply connected smooth 4-manifolds admit infinitely many distinct smooth structures. The knot surgery technique is following: Suppose that X is a simply connected smooth 4-manifold containing an embedded torus T of square 0. Then, for any knot $K \subset S^3$, one can construct a new 4-manifold, called *a knot surgery* 4*-manifold*,

$$X_K = X\sharp_{T=T_m}(S^1 \times M_K) \tag{1}$$

by taking a fiber sum along a torus T in X and $T_m = S^1 \times m$ in $S^1 \times M_K$, where M_K is the 3-manifold obtained by doing 0-framed surgery along K and m is the meridian of K. For example, if X is a simply connected elliptic surface $E(n)$, T is the elliptic fiber, and K is a fibered knot, then the knot surgery 4-manifold $E(n)_K$ admits not only a symplectic structure but also a genus $2g(K) + n - 1$ Lefschetz fibration structure.

In this article we investigate Lefschetz fibration structures on knot surgery 4-manifold $E(n)_K$ and we answer the following question proposed by I. Smith:

> Does the diffeomorphism type of a smooth 4-manifold determine the equivalence class of a Lefschetz fibration by curves of some given genus?

This is a joint work with Ki-Heon Yun.