Can general relativistic computers break the Turing barrier?

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Abstract. - Can general relativistic computers break the Turing barrier? - Are there final limits to human knowledge? - Limitative results versus human creativity (paradigm shifts). - Gödel's logical results in comparison/combination with Gödel's relativistic results. - Can Hilbert's programme be carried through after all?

1 Aims, perspective

The Physical Church-Turing Thesis, PhCT, is the conjecture that whatever physical computing device (in the broader sense) or physical thought experiment will be designed by any future civilization, it will always be simulatable by a Turing machine. The PhCT was formulated and generally accepted in the 1930's. At that time a general consensus was reached declaring PhCT valid, and indeed in the succeeding decades the PhCT was an extremely useful and valuable maxim in elaborating the foundations of theoretical computer science, logic, foundation of mathematics and related areas. But since PhCT is partly a physical conjecture, we emphasize that this consensus of the 1930's was based on the physical world-view of the 1930's. Moreover, many thinkers considered PhCT as being based on mathematics + common sense. But "common sense of today" means "physics of 100 years ago". Therefore we claim that the consensus accepting PhCT in the 1930's was based on the world-view deriving from Newtonian mechanics. Einstein's equations became known to a narrow circle of specialists around 1920, but around that time the consequences of these equations were not even guessed at. The world-view of modern black hole physics was very far from being generally known until much later, until after 1980.

Our main point is that in the last few decades (well after 1980) there has been a major paradigm shift in our physical world-view. This started in 1970 by Hawking's and Penrose's singularity theorem firmly establishing black hole physics and putting general relativity into a new perspective. After that, discoveries and new results have been accelerating. About 10 years ago astronomers obtained firmer and firmer evidence for the existence of larger and larger more exotic black holes [16],[15] not to mention evidence supporting the assumption that the universe is not finite after all [18]. Nowadays the whole field is in a state of constant revolution. If the background foundation on which PhCT was based has changed so fundamentally, then it is desirable to re-examine the status and scope of applicability of PhCT in view of the change of our general world-picture.

A special feature of the Newtonian world-view is the assumption of an absolute time scale. Indeed, this absolute time has its mark on the Turing machine as a model for computer. As a contrast, in general relativity there is no absolute time. Kurt Gödel was particularly interested in the exotic behavior of time in general relativity (GR). Gödel [7] was the first to prove that there are models of GR to which one cannot add a partial order satisfying some natural properties of a "global time". In particular, in GR various observers at various points of spacetime in different states of motion might experience time radically differently. Therefore we might be able to speed up the time of one observer, say C (for "computer"), relatively to the other observer, say P (for "programmer"). Thus P may observe C computing very fast. The difference between general relativity and special relativity is (roughly) that in general relativity this speed-up effect can reach, in some sense, infinity assuming certain conditions are satisfied. Of course, it is not easy to ensure that this speed-up effect happens in such a way that we could utilize it for implementing some non-computable functions.

In [6], [13] we prove that it is consistent with Einstein's equations, i.e. with general relativity, that by certain kinds of relativistic experiments, future generations might find the answers to non-computable questions like the halting problem of Turing machines or the consistency of Zermelo Fraenkel set theory (the foundation of mathematics, abbreviated as ZFC set theory from now on). For brevity, we call such thought experiments *relativistic computers.* Moreover, the spacetime structure we assume to exist in these experiments is based in [6],[13] on huge slowly rotating black holes the existence of which is made more and more likely (almost certain) by recent astronomical observations [16],[15].

We are careful to avoid basing the beyond-Turing power of our computer on "side-effects" of the idealizations in our mathematical model/theory of the physical world. For example, we avoid relying on infinitely small objects (e.g. pointlike test particles, or pointlike bodies), infinitely elastic balls, infinitely (or arbitrarily) precise measurements, or anything like these. In other words, we make efforts to avoid taking advantage of the idealizations which were made when GR was set up. Discussing physical realizability and realism of our design for a computer is one of the main issues in [13].

2 An intuitive idea for how relativistic computers work

In this section we would like to illuminate the ideas of how relativistic computers work, without going into the mathematical details. The mathematical details are elaborated, among others, in [6], [8], [13]. To make our narrative more tangible, here we use the example of huge slowly rotating black holes for our construction of relativistic computers. But we emphasize that there are many more kinds of spacetimes suitable for carrying out essentially the same construction (these are called Malament-Hogarth spacetimes in the physics literature). So, relativistic computers are not tied to rotating black holes, there are other general relativistic phenomena on which they can be based. An example is anti-de Sitter spacetime which attracts more and more attention in explaining recent discoveries in cosmology. We chose rotating black holes because they provide a tangible example for illustrating the kind of reasoning underlying general relativistic approaches to breaking the “Turing barrier”. Astronomical evidence for their existence makes them an even more attractive choice for our didactic purposes.

Let us start out from the so-called Gravitational Time Dilation effect (GTD). The GTD is a theorem of relativity which says that gravity makes time run slow. More sloppily: gravity slows time down. Clocks that are deep within gravitational fields run slower than ones that are farther out. We will have to explain what this means, but before explaining it we would like to mention that GTD is not only a theorem of general relativity. This theorem, GTD, can be already proved in (an easily understandable logic-based version of) special relativity in such a way that we simulate gravity by acceleration [10], [11]. So one advantage of GTD is that actually why it is true can be traced down by using only the simple methods of special relativity. Another advantage of GTD is that it has been tested several times, and these experiments are well known. Roughly, GTD can be interpreted by the following thought experiment. Choose a high enough tower on the Earth, put precise enough (say, atomic) clocks at the bottom of the tower and the top of the tower, then wait enough time, and compare the readings of the two clocks. Then the clock on the top will run faster (show more elapsed time) than the one in the basement, at each time one carries out this experiment.

How could we use GTD for designing computers that compute more than Turing Machines can? In the above outlined situation, by using the gravity of the Earth, it is difficult to make practical use of GTD. However, instead of the Earth, we could choose a huge black hole. A black hole is a region of spacetime with so big “gravitational pull” that even light cannot escape from this region. There are several types of black holes, an excellent source is Taylor and Wheeler [17]. For our demonstration of the main ideas here, we will use huge, slowly rotating black holes. (These are called slow-Kerr in the physics literature.) These black holes have two so-called *event horizons*, these are bubble-like surfaces one inside the other, from which even light cannot escape (because of the gravitational pull of the black hole). See Figures 1, 2. As we approach the outer event horizon from far away outside the black hole, the gravitational “pull” of the black hole approaches infinity as we get closer and closer to the event horizon. This is rather different from the Newtonian case, where the gravitational pull also increases but remains finite even on the event horizon.[1] For a while from now on “event horizon” means “outer event horizon”.

Let us study observers suspended over the event horizon. Here, suspended means that the distance between the observer and the event horizon does not change. Equivalently, instead of suspended observers, we could speak about observers whose spaceship is hovering over the event horizon, using their rockets

[1] The event horizon also exists in the Newtonian case, namely, in the Newtonian case, too, the event horizon is the “place” where the escape velocity is the speed of light (hence even light cannot escape to infinity from inside this event horizon “bubble”).

for maintaining altitude. Assume one suspended observer H is higher up and another one, L, is suspended lower down. So, H sees L below him while L sees H above him. Now the gravitational time dilation (GTD) will cause the clocks of H run faster than the clocks of L. Moreover, they both agree on this if they are watching each other e.g. via photons. Let us keep the height of H fixed. Now, if we gently lower L towards the event horizon, this ratio between the speeds of their clocks increases. Moreover, as L approaches the event horizon, this ratio approaches infinity. This means that for any integer n, if we want H's clocks to run n times as fast as L's clocks, then this can be achieved by lowering L to the right position.

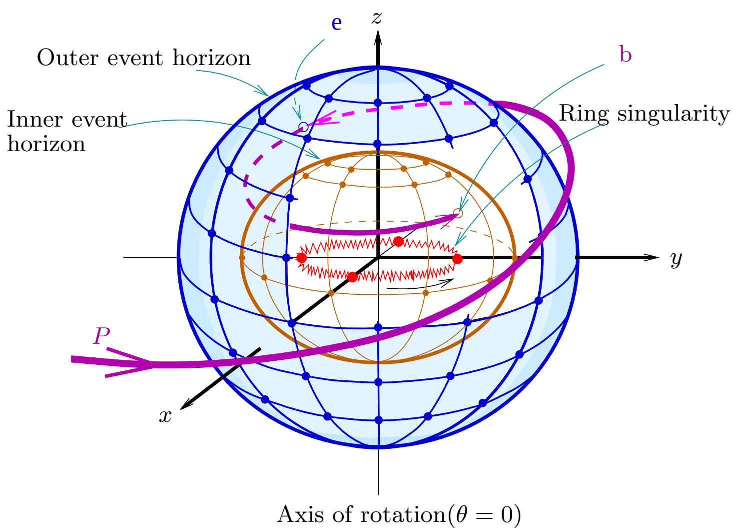

Fig. 1. A slowly rotating (Kerr) black hole has two event horizons and a ring-shape singularity (the latter can be approximated/visualized as a ring of extremely dense and thin "wire"). The ring singularity is inside the inner event horizon in the "equatorial" plane of axes x, y. Time coordinate is suppressed. Figure 2 is a spacetime diagram with x, y suppressed. Rotation of ring is indicated by an arrow. Orbit of in-falling programmer P is indicated, it enters outer event horizon at point e, and meets inner event horizon at point b.

Let us see what this means for computational complexity. If the programmer wants to speed up his computer with an arbitrarily large ratio, say n, then he can achieve this by putting the programmer to the position of L and putting the computer to the position of H. Already at this point we could use this arrangement with the black hole for making computers faster. The programmer goes very close to the black hole, leaving his computer far away. Then the programmer has to wait a few days and the computer does a few million year's job of computing and then the programmer knows a lot about the consequences of, say, ZFC set theory or whatever mathematical problem he is investigating. So we could use GTD for just speeding up computation which means dealing with complexity issues. However, we do not want to stop at complexity issues. Instead, we would like to see whether we can attack somehow the "Turing barrier".

The above arrangement for speeding the computer up raises the question of how the programmer avoids consequences of the fact that the whole manoeuver will slow down the programmer's own time relative to the time on his home planet, e.g. on the Earth. We will deal with this problem later. Let us turn now to the question of how we can use this effect of finite (but unbounded) speed-up to achieve an infinite speed-up, i.e. to breaking the Turing barrier.

If we could suspend the lower observer L on the event horizon itself then from the point of view of H, L's clocks would freeze, therefore from the point of view of L, H's clocks (and computers!) would run infinitely fast, hence we would have the desired infinite speed-up upon which we could then start our plan for breaking the Turing barrier. The problem with this plan is that it is impossible to suspend an observer on the event horizon. As a consolation for this, we can suspend observers arbitrarily close to the event horizon. To achieve an "infinite speed-up" we could do the following. We could lower and lower again L towards the event horizon such that L's clocks slow down (more and more, beyond limit) in such a way that there is a certain finite time-bound, say b, such that, roughly, throughout the whole history of the universe L's clocks show a time smaller than b. More precisely, by this we mean that whenever H decides to send a photon to L, then L will receive this photon before time b according to L's clocks. This is possible. See Figure 2.

Are we done, then? Not yet, there is a remaining task to solve. As L gets closer and closer to the event horizon, the gravitational pull or gravitational acceleration tends to infinity. If L falls into the black hole without using rockets to slow his fall, then he does not have to withstand the gravitational pull of the black hole. He would only feel the so-called tidal forces which can be made negligibly small by choosing a large enough black hole. However, his falling through the event horizon would be so fast that some photons sent after him by H would not reach him outside the event horizon. Thus L has to approach the event horizon relatively slowly in order that he be able to receive all possible photons sent to him by H. In theory he could use rockets for this purpose, i.e. to slow his fall (assuming he has unlimited access to fuel somehow). Because L approaches the event horizon slowly, he has to withstand this enormous gravity (or equivalently acceleration). The problem is that this increasing gravitational force (or acceleration) will kill L before his clock shows time b, i.e. before the planned task is completed.

At the outer event horizon of our black hole we cannot compromise between these two requirements by choosing a well-balanced route for L: no matter how he will choose his route, either L will be crashed by the gravitational pull, or some photons sent by H would not reach him. (This is the reason why we can not base our relativistic computer on the simplest kind of black holes, called

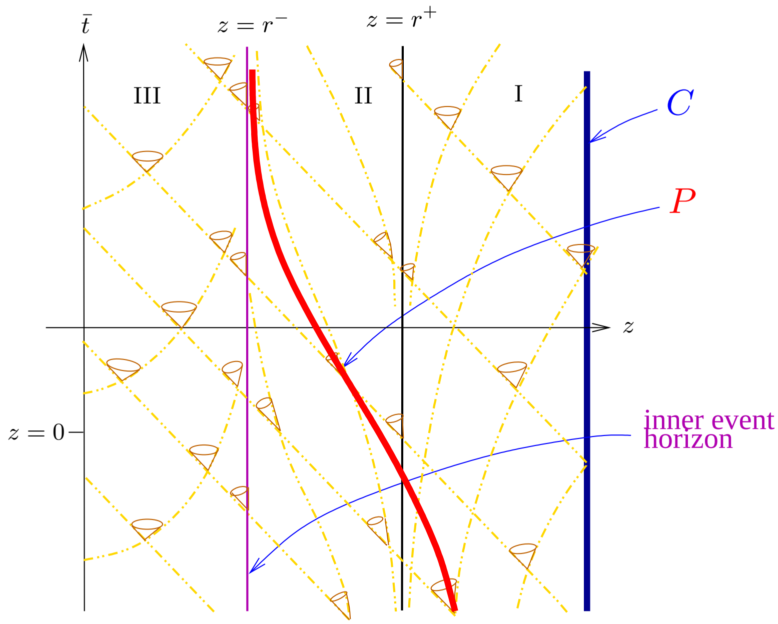

Fig. 2. The "t-z slice" of spacetime of slowly rotating black hole in coordinates where z is the axis of rotation of black hole. The pattern of light cones between the two event horizons r^- and r^+ illustrates that P can decelerate so much in this region that he will receive outside of r^- all messages sent by C. r^+ is the outer event horizon, r^- is the inner event horizon, $z = 0$ is the "center" of the black hole as in Figure 1. The tilting of the light cones indicates that not even light can escape through these horizons. That there is an outward push counteracting gravity can be seen by the shape of the light-cones in region III (central region of the black hole). The time measured by P is finite (measured between the beginning of the experiment and the event when P meets the inner event horizon at b) while the time measured by C is infinite.

Schwarzschild ones, which have only one event horizon and that behaves as we described as above.)

To solve this problem, we would like to achieve slowing down the "fall" of L not by brute force (e.g. rockets), but by an effect coming from the structure of spacetime itself. In our slowly rotating black hole, besides the gravitational pull of the black hole (needed to achieve the time dilation effect) there is a counteractive repelling effect coming from the revolving of the black hole. This repelling effect is analogous to "centrifugal force" in Newtonian mechanics and will cause L to slow down in the required rate. So the idea is that instead of the rockets of L, we would like to use for slowing the fall of L this second effect coming from the rotation of the black hole. In some black holes with such a repelling force, and this is the case with our slowly rotating one, two event horizons form, see Figures 1,2. The outer one is the result of the gravitational pull and behaves basically like the event horizon of the simplest, so-called Schwarzschild hole, i.e. as described above. The *inner event horizon* marks the point where the repelling force overcomes the gravitational force. So inside the inner horizon, it is possible again to "suspend" an observer, say L, i.e. it becomes possible for L to stay at a constant distance from the center of the black hole (or equivalently from the event horizons).

Let us turn to describing how a slowly rotating black hole implements the above outlined ideas, and how it makes possible to realize our plan for "infinite speed-up". Figure 1 represents a slowly rotating huge Kerr black hole and Figure 2 represents its spacetime structure. As we said, there are two event horizons, the inner one surrounded by the outer one. The source of gravity of the black hole is a ring shaped singularity situated inside the inner horizon. The path of the in-falling observer L can be planned in such a way that the event when L reaches the inner horizon corresponds to the time-bound b (on the wristwatch of L) mentioned above before which L receives all the possible messages sent out by H. In Figures 1,2 the world-lines of L and H are denoted as P and C because we think of L as the programmer and we think of H as L's computer.

By this we achieved the infinite speed-up we were aiming for. This infinite speed-up is represented in Figure 2 where P measures a finite proper time between its separation from the computer C (which is not represented in the figure) and its touching the inner horizon at proper time b (which point also is not represented in Figure 2). It can be seen in the figure that whenever C decides to send a photon towards P, that photon will reach P before P meets the inner horizon. The above outlined intuitive plan for creating an infinite speed-up effect is elaborated in more concrete mathematical detail in [6], [13].

Let us see how we can use all this to create a computer that can compute tasks which are beyond the Turing limit. Let us choose the task, for an example, to decide whether ZFC set theory is consistent. I.e. we want to learn whether from the axioms of set theory one can derive the formula FALSE. (This formula FALSE can be taken to be $\exists x(x \neq x)$.) The programmer P and his computer C are together (on Earth), not moving relative to each other, and P uses a finite time-period for transferring input data to the computer C as well as for programming C. After this, P boards a huge spaceship, taking all his mathematical friends with him, and chooses an appropriate route towards our huge slowly rotating black hole, entering the inner event horizon when his wrist-watch shows time b. While he is on his journey towards the black hole, the computer checks one by one the theorems of set theory, and as soon as the computer finds a contradiction in set theory, i.e. a proof of the formula FALSE, from the axioms of set theory, the computer sends a signal to the programmer indicating that set theory is inconsistent. (This is a special example only. The general idea is that the computer enumerates a recursively enumerable set and, before starting the computer, the programmer puts on the tape of the computer the name of the element which he wants to be checked for belonging to the set. The computer will search and as soon as it finds the element in question inside the set, the computer sends a signal.) If it does not find the thing in the set, the computer does nothing.

What happens to the programmer P from the point of view of the computer C? This is represented in Figure 2. Let C's coordinate system be the one represented in Figure 2. By saying "from the point of view of C" we mean "in this particular coordinate system (adjusted to C) in Fig.2". In this coordinate system when the programmer goes closer and closer to the inner horizon of the black hole, the programmer's clock will run slower and slower and slower, and eventually on the inner event horizon of the black hole the time of the programmer stops. Subjectively, the programmer does not experience it this way, this is how the computer will coordinatize it in the distance, or more precisely, how the coordinate system shown in Figure 2 represents it. If the computer thinks of the programmer, it will see in its mind's eye that the programmer's clocks stop and the programmer is frozen motionless at the event horizon of the black hole. Since the programmer is frozen motionless at the event horizon of the black hole, the computer has enough time to do the computation, and as soon as the computer has found, say, the inconsistency in set theory, the computer can send a signal and the computer can trust that the programmer—still with his clock frozen—will receive this signal before it enters the inner event horizon.

What will the programmer experience? This is represented in Figure 1. The programmer will see that as he is approaching the inner event horizon, his computer in the distance is running faster and faster and faster. Then the programmer falls through the inner event horizon of the black hole. If the black hole is enormous, the programmer will feel nothing when he passes either event horizon of the black hole—one can check that in case of a huge black hole the so-called tidal forces on the event horizons of the black hole are negligibly small [14]. So the programmer falls into the inner event horizon of the black hole and either the programmer will experience that a light signal arrives from the direction of the computer, of an agreed color and agreed pattern, or the programmer will observe that he falls in through the inner event horizon and the light signal does not arrive. After the programmer has crossed the inner event horizon, the programmer can evaluate the situation. If a signal arrives from the computer, this means that the computer found an inconsistency in ZFC set theory, therefore the programmer will know that set theory is inconsistent. If the light signal does not arrive, and the programmer is already inside the inner event horizon, then he will know that the computer did not find an inconsistency in set theory, did not send the signal, therefore the programmer can conclude that set theory is consistent. So he can build the rest of his mathematics on the secure knowledge of the consistency of set theory.

The next question which comes up naturally is whether the programmer can use this new information, namely that set theory is consistent, or whatever he wanted to compute, for his purposes. A pessimist could say that OK they are inside a black hole, so—now we are using common sense, we are not using relativity theory—common sense says that the black hole is a small unfriendly area and the programmer will sooner or later fall into the middle of the black hole where there is a singularity and the singularity will kill the programmer. The reason why we chose our black hole to be a huge slowly rotating one, say of mass $10^{10}m_\odot$, is the following. If the programmer falls into a black hole which is as big as this and it rotates slowly, then the programmer will have quite a lot of time inside the black hole because the center of the black hole is relatively far from the event horizon. But this is not the key point. If it rotates, the "matter content", the so-called singularity, which is the source of the gravitational field of the black hole so-to-speak, is not a point but a ring. So if the programmer chooses his route in falling into the black hole in a clever way, say, relatively close to the north pole instead of the equatorial plane, then the programmer can comfortably pass through the middle of the ring, never get close to the singularity and happily live on forever. We mean, the rules of relativity will not prevent him from happily living forever. He may have descendants, he can found society, he can use the so obtained mathematical knowledge.

Technical details of realizability of this general plan are checked in [13], [6]. The above outlined train of thought can be pushed through to show that any recursively enumerable set can be decided by a relativistic computer [6]. Actually, more than that can be done by relativistic computers, but it is not the purpose of the present paper to check these limits. These limits are addressed in [8], [9], [19].

3 Conclusion

A virtue of the present research direction is that it establishes connections between central questions of logic, foundation of mathematics, foundation of physics, relativity theory, cosmology, philosophy, particle physics, observational astronomy, computer science and AI [19]. E.g. it gives new kinds of motivation to investigating central questions of these fields like "is the universe finite or infinite (both in space and time) and in what sense", "exactly how do Kerr black holes evaporate" (quantum gravity), "how much matter is needed for coding one bit of information (is there such a lower bound at all)", questions concerning the statuses of the various cosmic censor hypotheses, questions concerning the geometry of rotating black holes [4], to mention only a few. The interdisciplinary character of this direction was reflected already in the 1987 course given by the present authors [12] during which the idea of relativistic hypercomputers emerged and which was devoted to connections between the above mentioned areas. Tangible data underlying the above interconnections and also more history, references are available in [13]. The book Earman [5, p.119, section 4.9] regards the same interdisciplinary perspective as described above to be one of the main virtues of the present research direction. It is the unifying power of logic which makes it viable to do serious work on such a diverse collection of topics. One of the main aims of the research direction represented by [3], [2], [1], [10], [11] is to make relativity theory accessible for anyone familiar with logic.

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