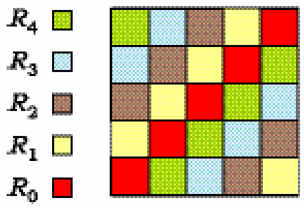
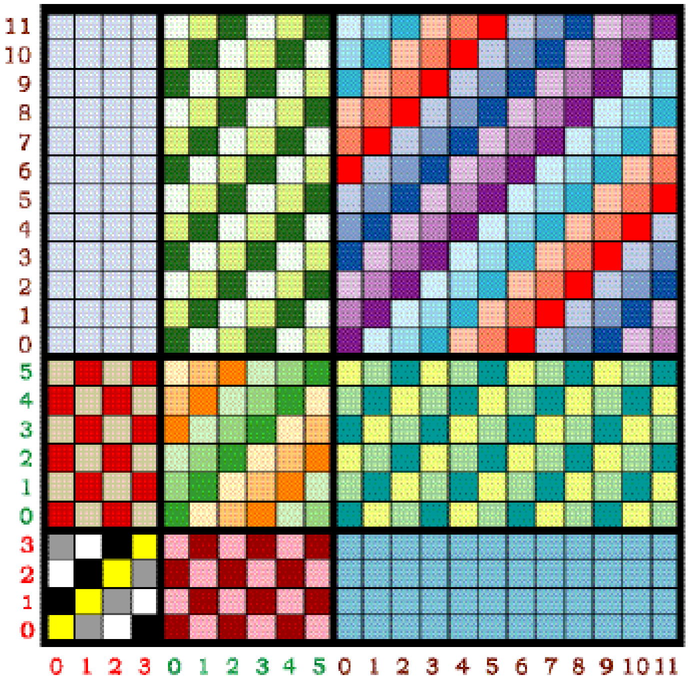


FIGURE 1. The Cayley representation of (the atoms of) $\mathfrak{Cm}(Z_5)$.

Cayley representation of the cyclic group Z_5. The little squares of the 5 by 5 big square represent pairs of form ‹i,j› with $0 \leq i,j < 5$. Little squares of the same color together form a binary relation. With the convention that we start numbering the little squares from the left bottom corner, the relations of color red, green,blue, brown, yellow represent addition of 0,1,2,3,4 respectively, modulo 5.

These relations form a partition of the big square. If we consider the operations „union" and "complementation" besides "relation composition" and "relation inversion" on the union of the elements of the partition, we get a so-called relation algebra.



This is a bigger relation algebra which was put together from Cayley representations of groups. The big square is divided by bold black lines to 9 *subsquares*. The three subsquares on the diagonal contain the Cayley representations of the cyclic groups Z_4, Z_6 and Z_{12}. It can be proved that when on the diagonal subsquares we have Cayley representations, the side-subsquares must also be Cayley-representations of quotient groups of the groups on the corresponding diagonal subsquares. More on these group-relation algebras can be found in [Groups and algebras of relations, Bulletin of Symbolic Logic 8,1 (2002), 38-64. by Andreka, H. and Givant, S].

One can construct relation algebras out of *geometries*, too, and then use them for building bigger relation algebras. This is illustrated in the next few figures.

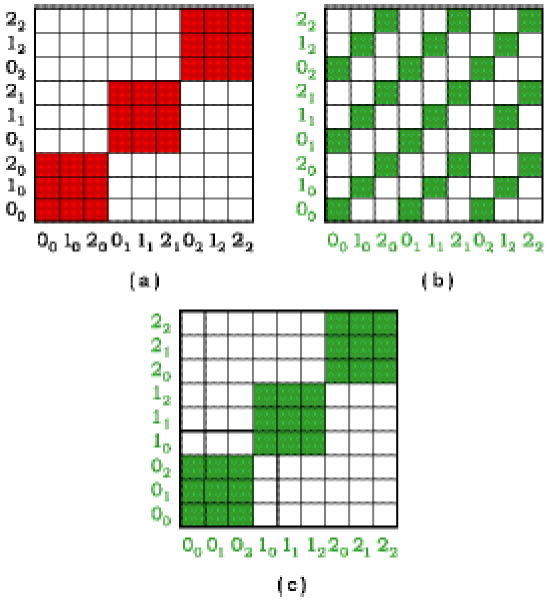


FIGURE 5. The affine representation of two equivalence elements. Figure (a) represents the element E^+_{01} in $\mathfrak{Cm}(P_0)/E^+_{01}$, and Figure (b) the element E^+_{10} in $\mathfrak{Cm}(P_1)/E^+_{10}$, under the original enumeration of the base set. Figure (c) represents the element E^+_{10} under a permuted enumeration of the base set.

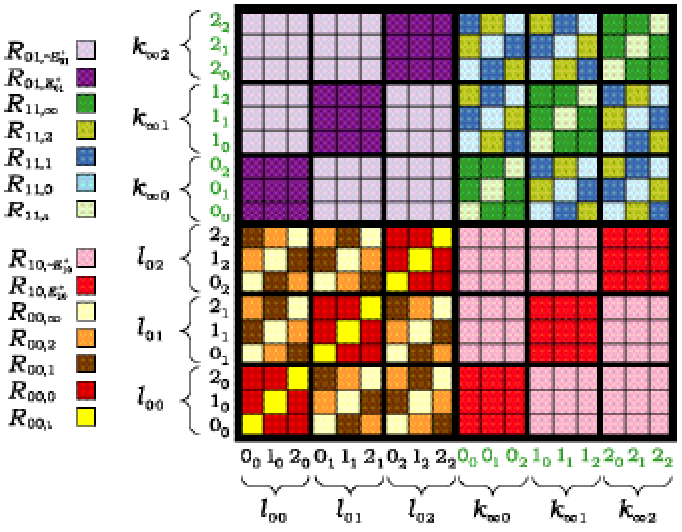


FIGURE 6. A representation of the geometric relation algebra over a frame with two projective lines of order three. In the lower left and upper right are the representations of two complex algebras of projective lines of order three. In the upper left and lower right are the representations of the equijections (the darker diagonal portions) and their complements relative to the local units.

The next algebra has similar abtract algebraic structure to the previous one. However, the side subsquares cannot be represented properly because they represent bijections between the sub-sides which have to be of size 9 and 25 in order to carry the representations of the two geometries on the diagonal sub-squares.

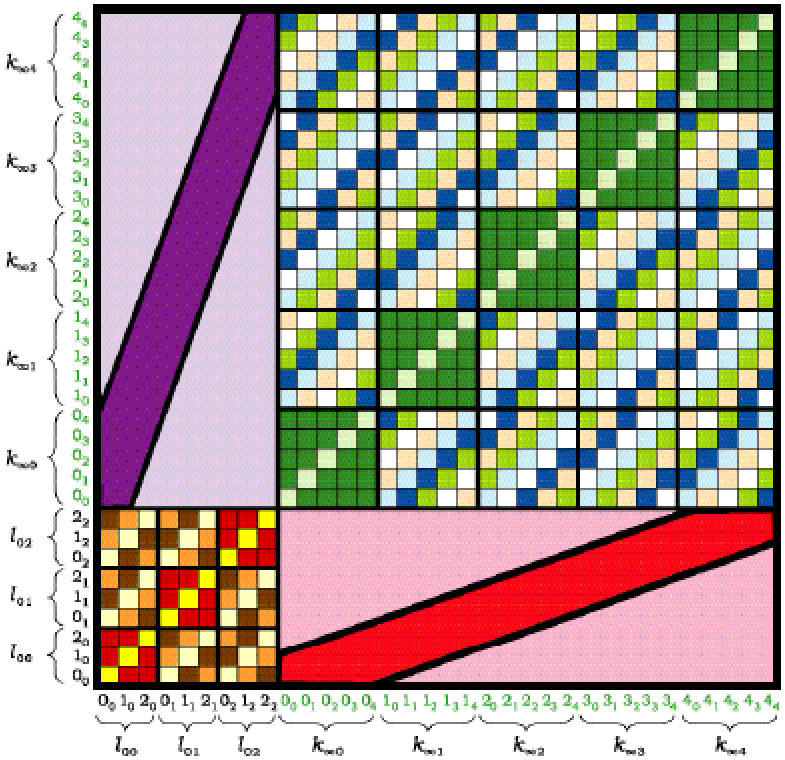


FIGURE 7. A non-representable geometric relation algebra over a frame with two projective lines, one of order three, the other of order five. In the lower left and upper right are the representations of the complex algebras of projective lines of orders three and five respectively. In the upper left and lower right are non-representable rectangular portions: each rectangle consists of an equijection and its complement relative to the local unit.

One can put together big relation algebras from small ones in various ways by putting relation algebras on the diagonal subsquares and then filling out the side subsquares in some pre-given ways. Conversely, one can use these constructions for breaking up big relation algebras into smaller ones. The usual algebraic methods give us tools for decomposing non-simple algebras into simple ones. The method represented here gives a tool for decomposing simple algebras into even smaller ones, and so for analyzing the structure of simple algebras. A study of hese tools together with a general structure theorem can be found in [The construction and analysis of simple relation algebras. Manuscript, xxiv+761 pp., Givant, S. with the collaboration of H. Andreka].

When constructing bigger algebras, we can either leave the side-squares empty, or we can repeat on the side subsquares a factor-algebra of the ones on the diagonal subsquares, or even we can insert more complex algebras into a subsquare. These and similar constructing methods are shown in the next figures.

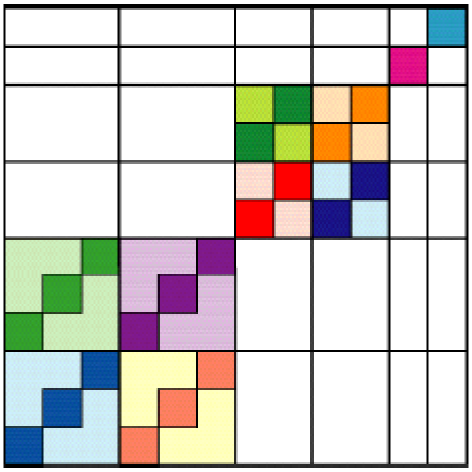


FIGURE 3. A simple, elementary relation algebra: a simple bijection semiproduct of $\mathfrak{M}_3$ to the power two (lower left), $\mathfrak{M}_2$ to the power two (middle), and $\mathfrak{M}_1$ to the power two (upper right).

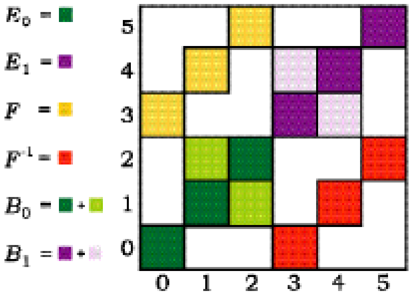


FIGURE 1. An illustration that the closure condition is necessary in Decomposition Theorem 12.22.

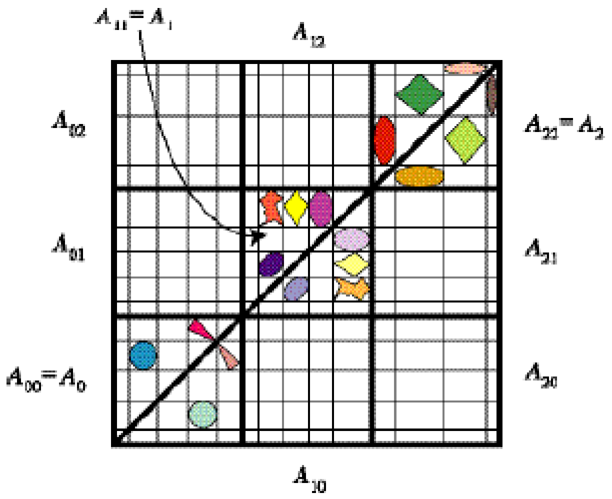


FIGURE 1. Sample elements of the subalgebra generated by a basic diagonal system.

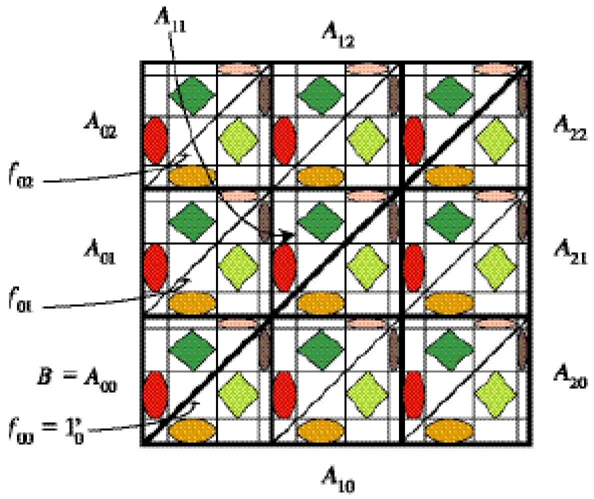


FIGURE 1. The subalgebra generated by a basic bijection system of power three.

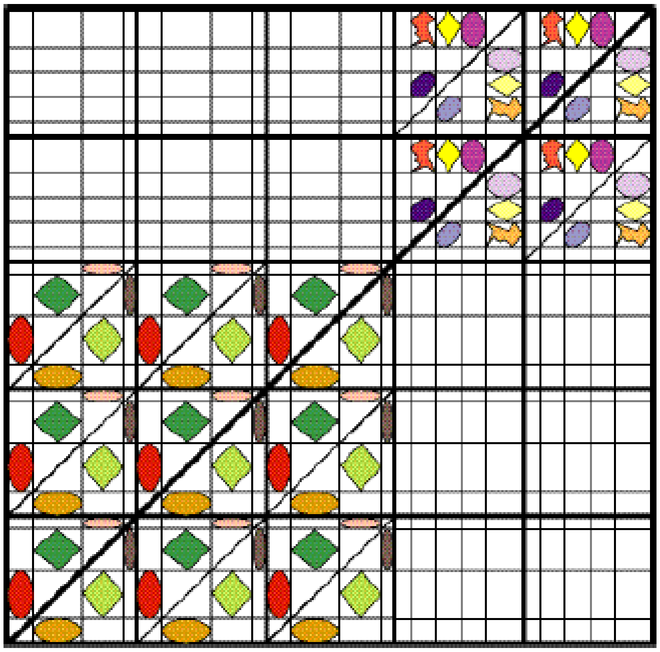


FIGURE 1. The subalgebra generated by a simple bijection system with two base algebras, one raised to the power three, the other raised to the power two.

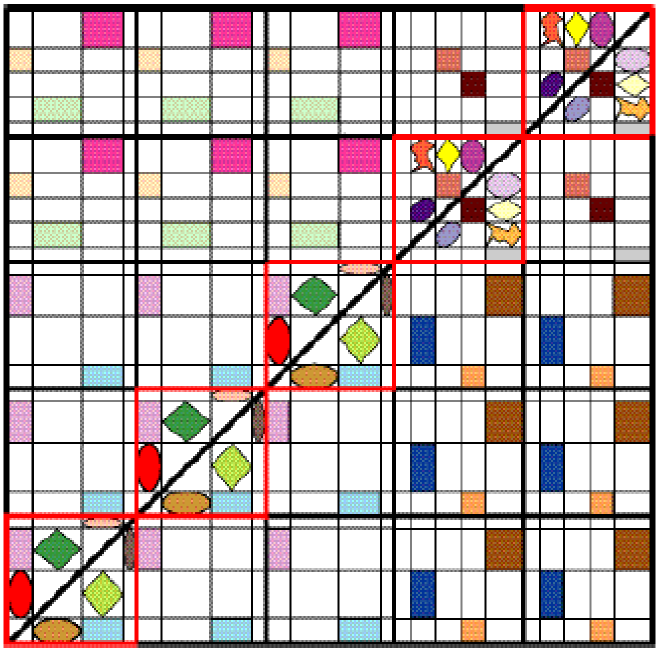


FIGURE 1. A simple closure of a set relation algebra $\mathfrak{B}$ with two ideal elements. The unit of $\mathfrak{B}$ — call it e — consists of the diagonal squares outlined in red. It is the sum of two ideal elements: the lower three squares form an ideal element of measure three, and the upper two form an ideal element of measure two. The universe of $\mathfrak{B}$ consists of certain subrelations of e. Objects of the same color are part of the same relation. The coequivalence rectangles (intersections of rectangles with $-e$) are part of the simple closure. (Not all rectangles are shown.)

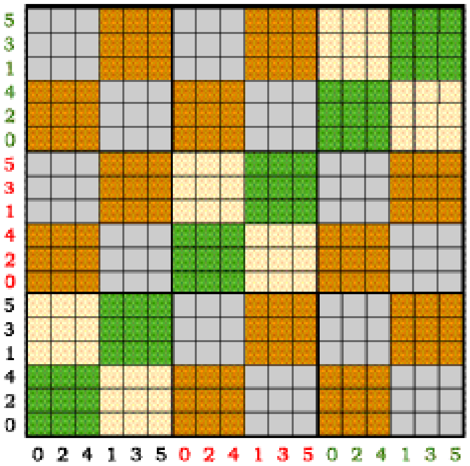


FIGURE 4. The expansion ϑ of the representation ρ, from Figure 2, of the algebra $\mathfrak{B}$.

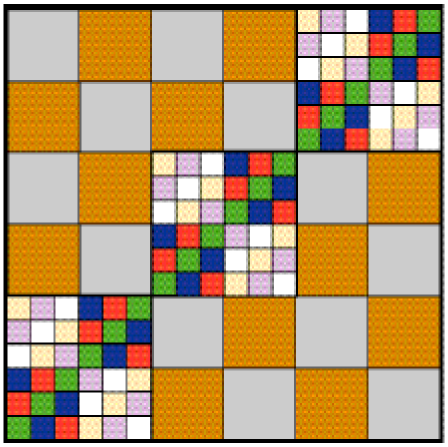


FIGURE 3. The result of inserting the group complex algebra $\mathfrak{C}$ for $\mathfrak{B}(e)$ in the integral relation algebra $\mathfrak{B}$.