

# List of axioms and axiom systems

oldalszámokat  
ellenőrizni!

Convention: In this list the axiom systems (i.e. theories) to be recalled will be boxed in. The only purpose of this is to make searching in the list easier.

## (1) Main axiom systems

**Basax**  $\stackrel{\text{def}}{=} \{ \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3}, \mathbf{Ax4}, \mathbf{Ax5}, \mathbf{Ax6}, \mathbf{AxE} \}$  (cf. p.51), where:

**Ax1**  $G = \text{Eucl}(n, \mathbf{F})$ , p.45.

**Ax2**  $\text{Obs} \cup \text{Ph} \subseteq \text{Ib}$ , p.48.

**Ax3**  $(\forall h \in \text{Ib})(\forall m \in \text{Obs}) \text{tr}_m(h) \in G$ , p.48.

**Ax4**  $(\forall m \in \text{Obs}) \text{tr}_m(m) = \bar{t}$ , p.48.

**Ax5**  $(\forall m \in \text{Obs})(\forall \ell \in G) \left( \text{ang}^2(\ell) < 1 \Rightarrow (\exists k \in \text{Obs}) \ell = \text{tr}_m(k) \text{ and} \right.$   
 $\left. \text{ang}^2(\ell) = 1 \Rightarrow (\exists ph \in \text{Ph}) \ell = \text{tr}_m(ph) \right)$ , p.50.

**Ax6**  $(\forall m, k \in \text{Obs}) \text{Rng}(w_m) = \text{Rng}(w_k)$ , p.50.

**AxE**  $(\forall m \in \text{Obs})(\forall ph \in \text{Ph}) v_m(ph) = 1$ , p.51.

**Newbasax**  $\stackrel{\text{def}}{=} (\mathbf{Basax} \setminus \{ \mathbf{Ax6}, \mathbf{Ax3}, \mathbf{AxE} \}) \cup \{ \mathbf{Ax6}_{00}, \mathbf{Ax6}_{01}, \mathbf{Ax3}_0, \mathbf{AxE}_0 \} =$   
 $\{ \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3}_0, \mathbf{Ax4}, \mathbf{Ax5}, \mathbf{Ax6}_{00}, \mathbf{Ax6}_{01}, \mathbf{AxE}_0 \}$  (cf. p.191), where:

**Ax6<sub>00</sub>**  $(\forall m, k \in \text{Obs}) w_m[\text{tr}_m(k)] \subseteq \text{Rng}(w_k)$ , p.190.

Intuitively, observer  $k$  sees all those events which are seen by another observer  $m$  on  $k$ 's life-line.

**Ax6<sub>01</sub>**  $(\forall m, k \in \text{Obs}) \text{Dom}(f_{mk}) \in \text{Open}$ , p.190.

**Ax3<sub>0</sub>**  $(\forall h \in \text{Ib}) (\text{tr}_m(h) \in G \cup \{ \emptyset \} \wedge (\exists k \in \text{Obs}) \text{tr}_k(h) \neq \emptyset)$ , p.191.

**AxE<sub>0</sub>**  $(\forall m \in \text{Obs})(\forall ph \in \text{Ph})(\text{tr}_m(ph) \neq \emptyset \Rightarrow v_m(ph) = 1)$ , p.191.

**Bax**  $\stackrel{\text{def}}{=} (\text{Newbasax} \setminus \{ \text{Ax5}, \text{AxE}_0 \}) \cup \{ \text{Ax5}^{\text{Obs}}, \text{Ax5}^{\text{Ph}}, \text{AxE}_{00}, \text{AxE}_{01} \} = \{ \text{Ax1}, \text{Ax2}, \text{Ax3}_0, \text{Ax4}, \text{Ax5}^{\text{Obs}}, \text{Ax5}^{\text{Ph}}, \text{Ax6}_{00}, \text{Ax6}_{01}, \text{AxE}_{00}, \text{AxE}_{01} \}$  (cf. p.219), where:

**Ax5<sup>Obs</sup>**  $(\exists ph)(\forall \ell) \left( m \overset{\circ}{\rightarrow} ph \wedge [\text{ang}^2(\ell) < v_m(ph) \Rightarrow (\exists k)\ell = \text{tr}_m(k)] \right)$ , p.218

**Ax5<sup>Ph</sup>**  $\text{ang}^2(\ell) = v_m(ph) \Rightarrow (\exists ph)\ell = \text{tr}_m(ph)$ , p.219.

**AxE<sub>00</sub>**  $(m \overset{\circ}{\rightarrow} ph_1, ph_2) \Rightarrow v_m(ph_1) = v_m(ph_2)$ , p.218.

**AxE<sub>01</sub>**  $v_m(ph) \neq 0$ , p.218.

**Flxbasax**  $\stackrel{\text{def}}{=} \text{Bax} + \text{AxE}_{02} = \{ \text{Ax1}, \text{Ax2}, \text{Ax3}_0, \text{Ax4}, \text{Ax5}^{\text{Obs}}, \text{Ax5}^{\text{Ph}}, \text{Ax6}_{00}, \text{Ax6}_{01}, \text{AxE}_{00}, \text{AxE}_{01}, \text{AxE}_{02} \}$  (cf. p.428), where:

**AxE<sub>02</sub>**  $(\forall m, k \in \text{Obs})(\forall ph, ph_1 \in \text{Ph})$   
 $[(m \overset{\circ}{\rightarrow} ph \wedge k \overset{\circ}{\rightarrow} ph_1) \Rightarrow v_m(ph) = v_k(ph_1)]$ , p.427.

**Bax<sup>-</sup>**  $\stackrel{\text{def}}{=} (\text{Bax} \setminus \{ \text{Ax5}^{\text{Obs}}, \text{Ax5}^{\text{Ph}}, \text{AxE}_{00} \}) \cup \{ \text{Ax5}_{\text{Obs}}, \text{Ax5}_{\text{Ph}}, \text{AxP1} \} = \{ \text{Ax1}, \text{Ax2}, \text{Ax3}_0, \text{Ax4}, \text{Ax5}_{\text{Obs}}, \text{Ax5}_{\text{Ph}}, \text{Ax6}_{00}, \text{Ax6}_{01}, \text{AxP1}, \text{AxE}_{01} \}$  (cf. p.479), where:

**AxP1** Intuitively, starting out from one point  $p$  of space-time, in every direction (forwards) there is at most one “speed of light” (i.e. photon-trace), formally:

$(\forall m \in \text{Obs})(\forall ph_1, ph_2 \in \text{Ph})(\forall d \in \text{directions})^{1298} \left( (ph_1 \text{ and } ph_2 \text{ are moving forwards in direction } d \text{ as seen by } m \text{ and } \text{tr}_m(ph_1) \cap \text{tr}_m(ph_2) \neq \emptyset) \Rightarrow \text{tr}_m(ph_1) = \text{tr}_m(ph_2) \right)$ , p.472.

**Ax5<sub>Ph</sub>** Intuitively, from any point  $p$  of space-time in any direction there is a photon moving forwards in that direction, cf. Fig.138 (p.477), formally:

$(\forall m \in \text{Obs})(\forall p \in {}^n F)(\forall d \in \text{directions})(\exists ph \in \text{Ph})$   
 $[p \in \text{tr}_m(ph) \wedge (ph \text{ is moving forwards in direction } d \text{ as seen by } m)]$ ,  
p.477.

---

<sup>1298</sup>Let us recall that directions are (nonzero) space-vectors, i.e.  $\text{directions} = {}^{n-1}F \setminus \{\bar{0}\}$ , cf. p.470.

**Ax5<sub>Obs</sub>** Intuitively: Let us fix an observer  $m$ , a direction  $d$ , and a point  $p$  of space-time. We will speak about things moving forwards in direction  $d$  through point  $p$  as seen by  $m$  (without mentioning all this data). Assume there is a photon moving in direction  $d$ . Then there is a photon in the same direction which is limiting in the following sense: For all speeds slower than this limiting photon, there is an observer moving with this speed, cf. Fig.139 (p.478). Formally:

$$\begin{aligned}
 & (\forall m \in Obs)(\forall p \in {}^n F)(\forall d \in \text{directions}) \\
 & \left( \left[ (\exists ph \in Ph)(p \in tr_m(ph) \wedge (ph \text{ is moving forwards in } d \text{ as seen by } m)) \right] \Rightarrow \right. \\
 & \left. \left[ (\exists ph \in Ph) \left( p \in tr_m(ph) \wedge (ph \text{ is moving forwards in } d \text{ as seen by } m) \wedge \right. \right. \right. \\
 & \left. \left. \left( \forall \lambda \in F \right) (0 \leq \lambda < v_m(ph) \Rightarrow (\exists k \in Obs)(p \in tr_m(k) \wedge v_m(k) = \lambda \wedge \right. \right. \\
 & \left. \left. (k \text{ is moving forwards in direction } d \text{ as seen by } m)) \right) \right] \right), \text{ p.477.}
 \end{aligned}$$

**Pax**  $\stackrel{\text{def}}{=} \{ \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3}_0, \mathbf{Ax4}, \mathbf{Ax5}_{Obs}^{--}, \mathbf{Ax6}_{00}, \mathbf{Ax6}_{01} \}$  (cf. p.482) where:

**Ax1, Ax2, Ax3<sub>0</sub>, Ax4, Ax6<sub>00</sub>, Ax6<sub>01</sub>** have already been listed.

**Ax5<sub>Obs</sub><sup>--</sup>** Intuitively, for each direction  $d$  there is a positive  $\lambda$  such that through any point there are observers moving forwards in direction  $d$  with all speeds smaller than  $\lambda$ . More precisely, for any observer  $m$  and for any plane  $P$  parallel with  $\bar{t}$  there is  $\lambda \in {}^+ F$  such that for any straight line  $\ell$  in  $P$ , with  $ang^2(\ell) < \lambda$ ,  $\ell$  is the trace of an observer (as seen by  $m$ , of course). In other words:

$$\begin{aligned}
 & (\forall m \in Obs)(\forall d \in \text{directions})(\forall p \in {}^n F)(\exists \lambda \in {}^+ F) \\
 & (\forall q \in {}^n F) \left[ \text{space}(p) - \text{space}(q) = \delta \cdot d \text{ for some } \delta \in F \Rightarrow (\forall 0 \leq \varepsilon < \lambda) \right. \\
 & \left. (\exists k \in Obs)(k \text{ moves forwards in direction } d \text{ with speed } \varepsilon \text{ and } q \in tr_m(k)) \right], \\
 & \text{p.481.}
 \end{aligned}$$

\* \* \*

Assume **Ax1, Ax2, Ax3<sub>0</sub>, AxP1**. Let  $m \in Obs$ . Then

$$c_m : {}^n F \times \text{directions} \xrightarrow{\circ} F \cup \{\infty\}$$

is a partial function such that  $c_m(p, d)$  is defined iff  $m$  sees a photon at point  $p$  moving forwards in direction  $d$ , and  $c_m(p, d)$  is the speed of this photon,<sup>1299</sup> cf. pp. 473, 535.

<sup>1299</sup>There is only one such speed because of **AxP1**.

§4-ben is ki-javítani az intuitív szöveget! esetleg a formulában is  $P$  síkot használni!

Let  $Th$  be one of our theories such that  $Th \models \{ \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3}_0, \mathbf{AxP1} \}$ . Then

$$\boxed{Th^\oplus} \stackrel{\text{def}}{=} Th + c_m(p, d) < \infty, \text{ p.643.}$$

Next, we turn to listing the Reichenbachian versions of our theories. For this we recall some notation.

Assume  $\mathbf{Bax}^-$ . By Thm.4.3.17 (p.488), the speed  $c_m(p, d)$  does not depend on  $p$ . This motivates the following:

$$c_m(d) \stackrel{\text{def}}{=} c_m(\bar{0}, d),$$

cf. p.488. Intuitively,  $c_m(d)$  is the (square of the) speed of light in direction  $d$  as seen by observer  $m$ .

speed of light  
vagy square of the  
speed of light?

Notation: Let  $m \in Obs$  and  $d \in directions$ . Then

$$T_m(d) \stackrel{\text{def}}{=} \begin{cases} 1/\sqrt{c_m(d)} & \text{if } 0 \neq c_m(d) < \infty, \\ \infty & \text{if } c_m(d) = 0, \\ 0 & \text{if } c_m(d) = \infty, \end{cases}$$

cf. p.555.  $T_m(d)$  is the reciprocal of the “speed of light”, i.e. it is the time needed for a photon to cover the unit distance in direction  $d$  (as seen by observer  $m$ ).

$$\boxed{\mathbf{Reich}_0(\mathbf{Bax})} \stackrel{\text{def}}{=} \mathbf{Bax}^- + \mathbf{R}(\mathbf{AxE}_{00}) \text{ (cf. p.562), where}$$

$$\mathbf{R}(\mathbf{AxE}_{00}) \text{ Ax}(\sqrt{\quad}) \text{ and} \\ (\forall d, d_1 \in directions)[T_m(d) + T_m(-d) = T_m(d_1) + T_m(-d_1)], \text{ p.557.}$$

$$\boxed{\mathbf{Reich}_0(\mathbf{Flxbasax})} \stackrel{\text{def}}{=} \mathbf{Bax}^- + \mathbf{R}(\mathbf{AxE}_{02}) \text{ (cf. p.562), where}$$

$$\mathbf{R}(\mathbf{AxE}_{02}) (\forall m, k \in Obs)(\forall d, d_1 \in directions) \\ T_m(d) + T_m(-d) = T_k(d_1) + T_k(-d_1), \text{ and Ax}(\sqrt{\quad}), \text{ p.557.}$$

$$\boxed{\mathbf{Reich}_0(\mathbf{Newbasax})} \stackrel{\text{def}}{=} \mathbf{Bax}^- + \mathbf{R}(\mathbf{AxE}) \text{ (cf. p.562), where}$$

$$\mathbf{R}(\mathbf{AxE}) \text{ Ax}(\sqrt{\quad}) \text{ and} \\ (\forall m \in Obs)(\forall d \in directions)T_m(d) + T_m(-d) = 2, \text{ p.557.}$$

$$\boxed{\mathbf{Reich}_0(\mathbf{Basax})} \stackrel{\text{def}}{=} \mathbf{Reich}_0(\mathbf{Newbasax}) + \mathbf{Ax6}, \text{ p.562.}$$

Let  $Th \in \{ \mathbf{Bax}, \mathbf{Flxbasax}, \mathbf{Newbasax}, \mathbf{Basax} \}$ . Then

$$\boxed{\mathbf{Reich}(Th)} \stackrel{\text{def}}{=} \mathbf{Reich}_0(Th) + \mathbf{R}_\Delta(E) \text{ (cf. p.576), where}$$

$$\mathbf{R}_\Delta(E) \quad (\forall m \in \text{Obs})(\exists r \in F)(\forall d_1, d_2, d_3 \in \text{directions}) \left[ d_1 + d_2 + d_3 = \bar{0} \Rightarrow \frac{|d_1| \cdot T_m(d_1) + |d_2| \cdot T_m(d_2) + |d_3| \cdot T_m(d_3)}{|d_1| + |d_2| + |d_3|} = r \right], \text{ p.574.}$$

## (2) Axioms concerning the direction of flow of time

The binary relation  $\uparrow \subseteq \text{Obs} \times \text{Obs}$  is defined as follows.

$$m \uparrow k \stackrel{\text{def}}{\iff} (\mathbf{f}_{km}(1_t)_t - \mathbf{f}_{km}(\bar{0})_t > 0), \text{ p.296.}$$

Intuitively,  $m \uparrow k$  means that  $m$  sees  $k$ 's clock running forwards. Further, if  $m, k \in \text{Obs}$  then  $m \text{ STL } k$  means that  $m$  sees  $k$  moving slower than light (cf. Def.4.2.6 on p.460).

$$\mathbf{Ax}(\uparrow) \quad (\forall m, m' \in \text{Obs}) (tr_m(m') = \bar{t} \Rightarrow m \uparrow m'), \text{ p.296.}$$

$$\mathbf{Ax}(\uparrow\uparrow) \quad (\forall m, k \in \text{Obs}) m \uparrow k, \text{ p.426.}$$

$$\mathbf{Ax}(\uparrow\uparrow_0) \quad (\forall m, k \in \text{Obs}) (m \stackrel{\circ}{\rightarrow} k \rightarrow m \uparrow k), \text{ p.840.}$$

$$\mathbf{Ax}(\uparrow\uparrow_{00}) \quad (\forall m, k \in \text{Obs}) (m \text{ STL } k \rightarrow m \uparrow k), \text{ p.840.}$$

## (3) Auxiliary axioms

Recall that

$$\text{Triv} = \{ f : f \text{ is an isometry of } {}^n\mathbf{F} \text{ and } f(1_t) - f(\bar{0}) = 1_t \},$$

cf. p.135.

$$\mathbf{Ax}(\text{Triv}) \quad (\forall m \in \text{Obs})(\forall f \in \text{Triv})(\exists k \in \text{Obs}) \mathbf{f}_{mk} = f, \text{ p.135.}$$

$$\mathbf{Ax}(\text{Triv}_t) \quad (\forall m \in \text{Obs})(\forall f \in \text{Triv}) \left( f[\bar{t}] = \bar{t} \Rightarrow (\exists k \in \text{Obs}) \mathbf{f}_{mk} = f \right), \text{ p.135.}$$

$\mathbf{Ax}(\text{Triv}_t)^-$  Assume we are given an observer  $m$  and a  $\text{Triv}$  transformation  $f$  that leaves the time-axis fixed. Then  $m$  has a brother, call it  $k$ , such that  $m$  thinks that (i) the coordinate axes of  $k$  are the  $f$ -images of the original coordinate axes  $\bar{x}_i$ , and (ii) the clock of  $k$  runs forwards, formally:

$$(\forall m \in \text{Obs})(\forall f \in \text{Triv}) [ f[\bar{t}] = \bar{t} \Rightarrow (\exists k \in \text{Obs})(\forall i \in n)(\mathbf{f}_{km}[\bar{x}_i] = f[\bar{x}_i] \wedge m \uparrow k) ], \text{ p.812.}$$

**Ax(||)**  $(\forall m, k \in Obs) \left( tr_m(k) \parallel \bar{t} \Rightarrow (f_{mk} \text{ is an isometry}) \right)$ , p.136.

**Ax(||)<sup>-</sup>**  $(\forall m, k \in Obs \cap Ib)$   
 $[tr_m(k) = \bar{t} \Rightarrow (f_{mk} = h \circ I, \text{ for some expansion } h \text{ and isometry } I)]$ ,  
p.828.

**Ax( $\sqrt{\quad}$ )**  $(\forall 0 < x \in F)(\exists y \in F) y^2 = x$ , p.91.

**Ax(rc)** (Axiom schema for real-closed fields)

**Ax( $\sqrt{\quad}$ )** +  $\{ \phi_{2n+1} : n \in \omega \}$ , where

$(\phi_n) \quad \forall x_0 \dots \forall x_n \exists y [x_n \neq 0 \rightarrow (x_0 + x_1 \cdot y + \dots + x_n \cdot y^n = 0)]$ ,

p.301.

**Ax(diswind)** (Axiom of disjoint windows)

$(\forall m, k \in Obs \cap Ib) [(m \overset{\circ}{\rightarrow} ph \wedge k \overset{\circ}{\rightarrow} ph) \Rightarrow m \overset{\circ}{\rightarrow} k]$ , p.812.

#### (4) Axioms concerning measuring distances

**Ax(eqtime)** Observers with common life-line agree on time-like distances, i.e.

$(\forall m, m' \in Obs)$   
 $\left( tr_m(m') = \bar{t} \Rightarrow (\forall p, q \in \bar{t}) |p - q| = |f_{mm'}(p) - f_{mm'}(q)| \right)$ , p.127.

**Ax(eqspace)** Observers agree on spatial distances, i.e.

$(\forall m, k \in Obs)(\forall p, q \in {}^n F)$   
 $\left( (p_t = q_t \wedge f_{mk}(p)_t = f_{mk}(q)_t) \Rightarrow |p - q| = |f_{mk}(p) - f_{mk}(q)| \right)$ ,  
p.136.

**Ax(eqm)** Inertial observers agree on distances, i.e.

$(\forall m, k \in Obs \cap Ib)(\forall i, j \in n)(\forall p, q \in \bar{x}_i)(\forall p', q' \in \bar{x}_j)$   
 $\left( [w_m(p) = w_k(p') \wedge w_m(q) = w_k(q')] \Rightarrow |p - q| = |p' - q'| \right)$ , p.796.

**(5) Axiom systems Pax<sup>+</sup>, Pax<sup>++</sup>, Pax<sub>+</sub><sup>+</sup>, Pax<sub>+</sub><sup>++</sup>, Wax, Wax<sup>+</sup>**

---

**Ax(Bw)**  $(\forall m, k \in Obs)[m \stackrel{\circ}{\rightarrow} k \Rightarrow (f_{mk} \text{ is betweenness preserving})]$ , p.1028.

**Ax♡**  $B = Obs \cup Ph$ , p.296.

**Ax( $\infty ph$ )**  $(\forall m \in Obs)(\forall ph, ph' \in Ph) \left( [\bar{0} \in tr_m(ph) \cap tr_m(ph')] \wedge (ph \text{ and } ph' \text{ move in the same direction as seen by } m) \wedge v_m(ph) = \infty \right] \rightarrow v_m(ph') = \infty \right)$ , p.1028.

sorrend?

Intuitively, no observer can emit simultaneously in the same direction two photons one with infinite speed and the other one with finite speed.

**Ax(ext)**  $(\forall m, k \in Obs) [w_m = w_k \Rightarrow m = k] \wedge (\forall b, b_1 \in B \setminus Obs)(\forall m \in Obs) [tr_m(b) = tr_m(b_1) \Rightarrow b = b_1]$ , p.298.

**Ax(Ph)**  $(\forall m \in Obs)(\forall p \in {}^nF)(\exists ph_1, ph_2 \in Ph) tr_m(ph_1) \cap tr_m(ph_2) = \{p\}$ , p.1073.

**Pax<sup>+</sup>**  $\stackrel{\text{def}}{=} \text{Pax} + \text{Ax}E_{01} + \text{Ax}(\text{Bw}) + \text{Ax}(\infty ph) + \left( [\text{Ax}(\text{eqtime}) \wedge (\forall m, k \in Obs)(\forall 0 < i \in \omega) tr_m(k) \neq \bar{x}_i] \vee \text{Ax}(\text{eqm}) \right)$ , p.1029.

**Pax<sup>++</sup>**  $\stackrel{\text{def}}{=} \text{Pax}^+ + \text{Ax}(\text{eqm}) + \text{Ax}(\text{ext}) + \text{Ax}\heartsuit$ , p.1081.

**Pax<sub>+</sub><sup>+</sup>**  $\stackrel{\text{def}}{=} \text{Pax}^+ + \text{Ax}(\text{diswind})$ , p.1086.

**Pax<sub>+</sub><sup>++</sup>**  $\stackrel{\text{def}}{=} \text{Pax}^{++} + \text{Ax}(\text{diswind})$ , p.1093.

**Wax**  $\stackrel{\text{def}}{=} \{ \text{Ax1}, \text{Ax2}, \text{Ax3}, \text{Ax4}, \text{Ax6}, \text{Ax}(\text{Bw}), \text{Ax}(Ph) \}$ , p.1073.

**Wax<sup>+</sup>**  $\stackrel{\text{def}}{=} \text{Wax} + \text{Ax}(\text{ext}) + \text{Ax}\heartsuit + \text{Ax}(\infty ph) + (\forall m, k)(f_{mk} \in \text{Afr})$ , p.1081.

## (6) Symmetry axioms

**Ax(symm<sub>0</sub>)**  $(\forall m, k \in Obs)(\exists m', k' \in Obs)$   
 $\left( tr_m(m') = tr_k(k') = \bar{t} \quad \wedge \quad f_{mk} = f_{k'm'} \right)$ , p.124.

**Ax(symm)** is defined to be **Ax(symm<sub>0</sub>)** + **Ax(eqtime)**, p.127.

**Ax(syt<sub>0</sub>)**  $(\forall m, k \in Obs) \left( tr_m(k) \neq \emptyset \Rightarrow \right.$   
 $\left. (\forall p \in \bar{t}) |f_{mk}(p)_t - f_{mk}(\bar{0})_t| = |f_{km}(p)_t - f_{km}(\bar{0})_t| \right)$ , p.134.

**Ax(syt<sub>00</sub>)**  $(\forall m, k \in Obs) [f_{mk}(\bar{0}) = \bar{0} \Rightarrow |f_{mk}(1_t)_t| = |f_{km}(1_t)_t|]$ , p.138.

**Ax(syt)\***  $f_{mk}(\bar{0}) = \bar{0} \Rightarrow f_{mk}(1_t)_t = f_{km}(1_t)_t$ , p.721.

**Ax(syx)\***  $(m, k \text{ are in pre-standard configuration}^{1300}) \Rightarrow |f_{mk}(1_x)_x| = |f_{km}(1_x)_x|$ , p.725.

**Ax(speedtime)**  $(\forall m, k, m', k' \in Obs) \left( v_m(k) = v_{m'}(k') \Rightarrow \right.$   
 $\left. (\forall p \in \bar{t}) |f_{mk}(p)_t - f_{mk}(\bar{0})_t| = |f_{m'k'}(p)_t - f_{m'k'}(\bar{0})_t| \right)$ , p.137.

**Ax□1**  $(\forall m, k, m' \in Obs)(\exists k' \in Obs)f_{mk} = f_{m'k'}$ , p.350.

**Ax□2**  $(\forall m, k, m', k' \in Obs)(tr_m(k) = tr_{m'}(k') \rightarrow$   
 there is an isometry  $N$  of  ${}^n\mathbf{F}$  such that  $N[\bar{t}] \parallel \bar{t}$  and  $f_{mk} = f_{m'k'} \circ N$ ), p.350.

**Ax△1**  $(\forall m, k \in Obs)(\exists k' \in Obs)(tr_m(k) = tr_m(k') \wedge f_{mk'} = f_{k'm})$ , p.351.

**Ax△2**  $(\forall m, k \in Obs)$  (there is an isometry  $N$  of  ${}^n\mathbf{F}$  such that  
 $N[\bar{t}] \parallel \bar{t}$  and  $f_{mk} = N \circ f_{km} \circ N$ ), p.351.

**Ax( $\omega$ )<sup>0</sup>** is defined to be the disjunction of the following symmetry axioms:  
**Ax(syt<sub>0</sub>)**, **Ax(symm)**, **Ax(speedtime)**, **Ax△1+Ax(eqtime)**, **Ax△2**,  
**Ax□1+Ax(eqtime)**, **Ax□2**, p.844.

**Ax( $\omega$ )<sup>00</sup>** is defined to be the disjunction of the following symmetry axioms: **Ax( $\omega$ )<sup>0</sup>**,  
**Ax(eqspace)**, **Ax(eqm)+Ax(Triv<sub>t</sub>)<sup>-</sup>**, p.844.

**Ax( $\omega$ )<sup>#</sup>** is defined to be **Ax( $\omega$ )<sup>0</sup>+Ax(Triv<sub>t</sub>)<sup>-</sup>+Ax( $\sqrt{\quad}$ )**, p.844.

<sup>1300</sup> $m$  and  $k$  are said to be in *pre-standard configuration* iff  $f_{mk}(\bar{0}) = \bar{0}$  and  $f_{mk}[\text{Plane}(\bar{t}, \bar{x})] = \text{Plane}(\bar{t}, \bar{x})$ . Cf. Def.4.6.5 (p.602) and Fig.201 (p.603).

$\mathbf{Ax}(\omega)^{\#}$  is defined to be  $\mathbf{Ax}(\omega)^{00} + \mathbf{Ax}(Triv_t)^- + \mathbf{Ax}(\sqrt{\quad})$ , p.844.

$\mathbf{Ax}(\mathbf{symm})^\dagger$  is defined to be  $\mathbf{Ax}(\mathbf{symm}) + \mathbf{Ax}(Triv) + \mathbf{Ax}(\|)$ , p.151.

$\mathbf{Ax}(\omega) \mathbf{Ax}\square 1 \wedge \mathbf{Ax}\square 2 \wedge \mathbf{Ax}\triangle 1 \wedge \mathbf{Ax}\triangle 2$ , p.351.

$\mathbf{Ax}(\omega^-) \mathbf{Ax}\square 1 \vee \mathbf{Ax}\square 2 \vee \mathbf{Ax}\triangle 1 \vee \mathbf{Ax}\triangle 2$ , p.351.

### (7) Symmetry axioms adequate for Reichenbachian theories

$\mathbf{R}^+(\mathbf{Ax} \mathbf{eqsp})$  Intuitively, the thickness of spaceships do not change in the direction orthogonal to movement (cf. pp. 608–614), formally:

Assume  $m, k \in Obs$  such that  $m \overset{\circ}{\rightarrow} k$ . Assume  $P, Q$  are parallel planes of  ${}^nF$  such that they are parallel with both  $\bar{t}$  and  $tr_m(k)$ . Then

$$\begin{aligned} \text{Eudist}(P, Q) &= \text{Eudist}(f_{mk}[P], f_{mk}[Q]), \text{ p.614, where} \\ \text{Eudist}(H, H_1) &\stackrel{\text{def}}{=} \inf \{ \|p - q\| : p \in H \text{ and } q \in H_1 \}. \end{aligned}$$

$\mathbf{R}(\mathbf{Ax} \mathbf{eqsp})$  Intuitively, the thickness of spaceships do not change in the direction orthogonal to movement (cf. pp. 608–614), formally:

Assume  $m$  and  $k$  are in pre-standard configuration<sup>1301</sup>. Let  $P$  be a (2-dimensional) plane parallel with  $\text{Plane}(\bar{t}, \bar{x})$ . Then the distance between  $P$  and  $\text{Plane}(\bar{t}, \bar{x})$  is the same as the distance between  $f_{mk}[P]$  and  $f_{mk}[\text{Plane}(\bar{t}, \bar{x})]$ . Formally,

$$\begin{aligned} \text{Eudist}(P, \text{Plane}(\bar{t}, \bar{x})) &= \text{Eudist}(f_{mk}[P], f_{mk}[\text{Plane}(\bar{t}, \bar{x})]), \text{ p.611, where} \\ \text{Eudist}(H, H_1) &\stackrel{\text{def}}{=} \inf \{ \|p - q\| : p \in H \text{ and } q \in H_1 \}, \text{ cf. p.609.} \end{aligned}$$

See Fig.205 on p.611.

$\mathbf{R}(\mathbf{Ax} \mathbf{syt}_0)$  Intuitively  $m$  and  $k$  *literally* see, via photons, each other's clocks slowing down with the same rate, see Fig.207 (p.616), formally:

$$\begin{aligned} (\forall m, k \in Obs)[f_{mk}(\bar{0}) = \bar{0} \Rightarrow \\ (\forall p \in \bar{t}) |view_m(f_{km}(p))| = |view_k(f_{mk}(p))|] \text{ (cf. p.615),} \\ \text{where } view_m \stackrel{\text{def}}{=} \{ \langle p, q \rangle \in {}^nF \times \bar{t} : p_t \leq q_t \text{ and } (\exists ph \in Ph) p, q \in tr_m(ph) \}, \\ \text{cf. Fig.206 (p.615).} \end{aligned}$$

$\mathbf{R}(\mathbf{sym})$  is defined to be  $\mathbf{R}(\mathbf{Ax} \mathbf{eqsp}) + \mathbf{R}(\mathbf{Ax} \mathbf{syt}_0)$ , p.616.

---

<sup>1301</sup>Cf. footnote 1300 on p.1260 for the notion of a pre-standard configuration.

## (8) Twin paradox

Let  $m, k \in Obs$ . Then  $m$  *STL*  $k$  means that  $m$  sees  $k$  moving slower than light, cf. Def.4.2.6 on p.460 for details.

$$\mathbf{Ax}(\mathbf{TwP}) (\forall m, k_1, k_2 \in Obs)(\forall p, q, r \in {}^nF) \\ \left( [m \text{ STL } k_1 \wedge m \text{ STL } k_2 \wedge p_t < q_t < r_t \wedge \{p\} = tr_m(m) \cap tr_m(k_1) \wedge \{q\} = tr_m(k_1) \cap tr_m(k_2) \wedge \{r\} = tr_m(m) \cap tr_m(k_2)] \Rightarrow \right. \\ \left. |p_t - r_t| > |f_{mk_1}(p)_t - f_{mk_1}(q)_t| + |f_{mk_2}(q)_t - f_{mk_2}(r)_t| \right), \text{ p.460}$$

(cf. Fig.43 on p.141).

The *existential version*  $\mathbf{Ax}(\exists\mathbf{TwP})$  of the twin paradox is defined as follows. First, let us notice that  $\mathbf{Ax}(\mathbf{TwP})$  is a formula of the pattern

$$(\forall m \dots)(\forall p \dots) \left( [\dots] \Rightarrow |\dots| > |\dots| \right).$$

Let  $\psi_1, \psi_2$  be formulas such that  $\mathbf{Ax}(\mathbf{TwP})$  is the formula

$$(\forall m, k_1, k_2 \in Obs)(\forall p, q, r \in {}^nF)(\psi_1 \Rightarrow \psi_2).$$

§4.2  $\mathbf{Ax}(\exists\mathbf{TwP})$

def-ját

kijavítani:  $\forall p, q, r$  helyett  $\exists p, q, r$  kellene hogy legyen.

Now we define  $\mathbf{Ax}(\exists\mathbf{TwP})$  as follows.

$$\mathbf{Ax}(\exists\mathbf{TwP}) (\exists m, k_1, k_2 \in Obs)(\exists p, q, r \in {}^nF)[\psi_1 \wedge \psi_2], \text{ p.461.}$$

## (9) Axiom systems Specrel, Flxspecrel, BaCo, Compl, NewtK<sup>-</sup>, NewtK

$$\boxed{\text{Specrel}} \stackrel{\text{def}}{=} \mathbf{Basax} + \mathbf{Ax}(\text{symm})^\dagger, \text{ p.151.}$$

$$\boxed{\text{Flxspecrel}} \stackrel{\text{def}}{=} \mathbf{Bax} + \mathbf{Ax6} + \mathbf{Ax}(\text{symm})^\dagger + \mathbf{AxE}_{02}, \text{ p.428.}$$

$$\boxed{\text{Compl}} \stackrel{\text{def}}{=} \{ \mathbf{Ax}(\text{symm}), \mathbf{Ax}\heartsuit, \mathbf{Ax}(\uparrow), \mathbf{Ax5}^+, \mathbf{Ax}(\text{ext}), \mathbf{Ax}(\text{Triv}_t) \} \text{ (cf. p.298),}$$

where

$$\mathbf{Ax5}^+ \quad \ell \in \text{SlowEucl} \quad \Rightarrow \quad (\exists k \in Obs) (\ell = tr_m(k) \wedge m \uparrow k), \text{ p.297.}$$

$$\boxed{\text{BaCo}} \stackrel{\text{def}}{=} \mathbf{Basax} + \mathbf{Compl}, \text{ p.298.}$$

$$\boxed{\text{NewtK}^-} \stackrel{\text{def}}{=} \mathbf{Bax} + \mathbf{Ax6} + \mathbf{Ax}(\text{symm})^\dagger + (\forall m \in Obs)c_m = \infty \text{ (cf. p.426), where}$$

$c_m$  is the speed of light for observer  $m$ , assuming  $\mathbf{Bax}$ .

$$\boxed{\text{NewtK}} \stackrel{\text{def}}{=} \text{NewtK}^- + \mathbf{Ax}(\uparrow\uparrow) + \mathbf{Ax}\square\mathbf{1}, \text{ p.426.}$$

## (10) Geometrical axioms and axiom systems

Axioms **A<sub>0</sub>–A<sub>4</sub>** and **P<sub>1</sub>, P<sub>2</sub>, Pa** below apply to geometries with reducts  $\langle Mn; Bw \rangle$  or  $\langle Mn; coll \rangle$ . In the case of “ $\langle Mn; Bw \rangle$ ”  $coll$  is a defined relation, cf. p.818. The new universe (or sort) *lines* is (explicitly) defined over  $\langle Mn; coll \rangle$  on p.1037. For  $H \subseteq Mn$ ,  $Plane'(H)$  is intuitively the “ $n$ -long closure of  $H$  under  $coll$ ” (cf. Def.6.2.15 on p.819), where throughout  $n$  is the dimension of our geometry and  $n \geq 2$ .

**A<sub>0</sub>**  $(\forall a, b, c \in Mn)[coll(a, b, c) \leftrightarrow (\exists \ell \in lines) a, b, c \in \ell]$ , p.1038.

**A<sub>1</sub>**  $(\forall a, b \in Mn)(a \neq b \rightarrow (\exists! \ell \in lines) a, b \in \ell)$ , p.1038.

**A<sub>2</sub>** Intuitively, if  $H$  is a less than  $n + 2$  element subset of  $Mn$  then the “ $n$ -long closure”  $Plane'(H)$  of  $H$  under  $coll$  will be closed under  $coll$ , hence the plane  $Plane(H)$  generated by  $H$  coincides with  $Plane'(H)$  (cf. Def.6.2.15, p.819), formally:

$$(\forall H \subseteq Mn) \left( (|H| \leq n+1 \wedge a, b \in Plane'(H) \wedge coll(a, b, c)) \rightarrow c \in Plane'(H) \right), \text{ p.1039.}$$

**A<sub>3</sub>** Intuitively, if  $i \leq n$  and  $H$  is an  $i + 1$  element independent subset<sup>1302</sup> of  $Mn$  then there is exactly one  $i$ -dimensional plane<sup>1303</sup> that contains  $H$ , formally:

$$(\forall H, H' \subseteq Mn) \left( (|H| = |H'| \leq n+1 \wedge (\text{both } H \text{ and } H' \text{ are independent}) \wedge H \subseteq Plane'(H')) \rightarrow Plane'(H) = Plane'(H') \right), \text{ p.1039.}$$

**A<sub>4</sub>**  $Mn$  is an  $n$  dimensional plane, p.1039.

In connection with axioms **P<sub>1</sub>, P<sub>2</sub>** below we note that the relation of parallelism  $\parallel$  on *lines* is defined the usual way in Def.6.6.19 on p.1039.

**P<sub>1</sub>** (Euclid’s axiom)

$$(\forall \ell \in lines)(\forall a \in Mn)(\exists! \ell' \in lines)(a \in \ell' \wedge \ell \parallel \ell'), \text{ p.1040.}$$

**P<sub>2</sub>**  $(\ell \parallel \ell' \wedge \ell' \parallel \ell'') \rightarrow \ell \parallel \ell'', \text{ p.1040.}$

<sup>1302</sup>Let  $H \subseteq Mn$ .  $H$  is called *independent* iff  $(\forall e \in H) e \notin Plane'(H \setminus \{e\})$ , cf. p.1039.

<sup>1303</sup>Let  $P \subseteq Mn$ .  $P$  is called an  $i$ -dimensional iff there is an  $i + 1$  element independent subset  $H$  of  $Mn$  such that  $Plane'(H) = P$ , where for the notion of an independent subset cf. footnote 1302. Cf. Def.6.6.18(ii) on p.1039.

$\boxed{\text{ag}} \stackrel{\text{def}}{=} \{\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \mathbf{A}_4, \mathbf{P}_1, \mathbf{P}_2\}$ , p.1040. **ag** is the axiom system for affine geometries.

For  $a, b, c, d \in Mn$ , the abbreviation  $\langle a, b \rangle \parallel \langle c, d \rangle$  means that  $a \neq b$ ,  $c \neq d$ , and there are  $\ell, \ell' \in \text{lines}$  such that  $a, b \in \ell$ ,  $c, d \in \ell'$  and  $\ell \parallel \ell'$ , cf. p.1042.

**Pa** (Pappus-Pascal property)

$$(\forall \ell, \ell' \in \text{lines})(\forall a, b, c \in \ell \setminus \ell')(\forall a', b', c' \in \ell' \setminus \ell)$$

$$[(\langle a, b' \rangle \parallel \langle a', b \rangle \wedge \langle a, c' \rangle \parallel \langle a', c \rangle) \rightarrow \langle b, c' \rangle \parallel \langle b', c \rangle],$$

see Fig.319, p.1042.

$\boxed{\text{pag}} \stackrel{\text{def}}{=} \text{ag} + \mathbf{Pa}$ , p.1042. **pag** is the axiom system for Pappian affine geometries.

Axioms **B<sub>1</sub>–B<sub>3</sub>** below apply to geometries with reducts  $\langle Mn; Bw \rangle$ . (*coll* is a defined relation.)

**B<sub>1</sub>**  $Bw(a, b, c) \rightarrow (a \neq b \neq c \neq a \wedge Bw(c, b, a) \wedge \neg Bw(b, a, c))$ , p.1043.

**B<sub>2</sub>**  $a \neq b \rightarrow (\exists c)Bw(a, b, c)$ , p.1043.

**B<sub>3</sub>** (Pasch's Law)

Intuitively, if a line  $\ell$  lies in the plane determined by a triangle  $abc$ , and passes between  $a$  and  $b$  but not through  $c$ , then  $\ell$  passes between  $a$  and  $c$ , or between  $b$  and  $c$ , formally:

$$(\neg \text{coll}(a, b, c) \wedge \ell \subseteq \text{Plane}'(\{a, b, c\}) \wedge (\exists d \in \ell)Bw(a, d, b)) \rightarrow (\exists e \in \ell)(Bw(a, e, c) \vee Bw(b, e, c)), \text{ p.1043 (cf. Fig.320 on p.1044).}$$

$\boxed{\text{opag}} \stackrel{\text{def}}{=} \text{pag} + \{\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3\}$ , p.1044. **opag** is the axiom system for ordered Pappian affine geometries.

Axioms **L<sub>1</sub>, ..., L<sub>10</sub>** below apply to geometries with reducts

$$\langle Mn, L; L^T, L^{Ph}, L^S, \in, \prec, Bw, \perp_r, eq \rangle.$$

Further, *coll* is a defined relation and it is defined over  $\langle Mn; Bw \rangle$ , cf. p.818, and *lines* is the new sort first-order defined from *coll*.

**L<sub>1</sub>**  $L \subseteq \text{lines}$ , p.1071.

**L<sub>2</sub>**  $(\forall a \in Mn)(\exists \ell, \ell' \in L^{Ph}) \ell \cap \ell' = \{a\}$ , p.1071.

**lopag**  $\stackrel{\text{def}}{=} \text{opag} + \mathbf{L}_1 + \mathbf{L}_2$ , p.1071.

**L<sub>3</sub>**  $([a \prec b \wedge (Bw(a, b, c) \vee Bw(a, c, b))] \rightarrow a \prec c) \wedge$   
 $([a \prec b \wedge (Bw(c, a, b) \vee Bw(a, c, b))] \rightarrow c \prec b)$ , p.1076.

**L<sub>4</sub>** Intuitively,  $eq$  is (very) symmetric, formally:  
 $\langle a, b \rangle eq \langle c, d \rangle \rightarrow (\langle c, d \rangle eq \langle a, b \rangle \wedge \langle b, a \rangle eq \langle c, d \rangle \wedge \langle a, a \rangle eq \langle c, c \rangle)$ ,  
p.1076.

**L<sub>5</sub>**  $eq$  is transitive, i.e.  
 $(\langle a, b \rangle eq \langle c, d \rangle \wedge \langle c, d \rangle eq \langle e, f \rangle) \rightarrow \langle a, b \rangle eq \langle e, f \rangle$ , p.1076.

**L<sub>6</sub>** (For the intuitive meaning of this axiom see Fig.326 on p.1076.)  
 $(\forall \ell, \ell' \in L)(\forall o, e, e', a, a' \in Mn) ([\ell \cap \ell' = \{o\} \wedge e, a \in \ell \wedge e', a' \in \ell' \wedge$   
 $\langle e, e' \rangle \parallel \langle a, a' \rangle \wedge \langle o, e \rangle eq \langle o, e' \rangle] \rightarrow \langle o, a \rangle eq \langle o, a' \rangle)$ , p.1076.

**L<sub>7</sub>** (For the intuitive meaning of this axiom see Fig.327 on p.1077.)  
 $(\forall \ell \in L^T \cup L^S)(\forall a, b, c, d, e, f \in Mn) [(a, b, c, d \in \ell \wedge$   
 $\langle a, b \rangle \parallel \langle e, f \rangle \parallel \langle c, d \rangle \wedge \langle a, e \rangle \parallel \langle b, f \rangle \wedge \langle c, e \rangle \parallel \langle d, f \rangle) \rightarrow \langle a, b \rangle eq \langle c, d \rangle]$ ,  
p.1076.

**L<sub>8</sub>**  $\perp_r$  is symmetric, i.e.  
 $(\forall \ell, \ell' \in L) (\ell \perp_r \ell' \rightarrow \ell' \perp_r \ell)$ , p.1076.

**L<sub>9</sub>**  $\perp_r$  is closed under parallelism, i.e.  
 $(\forall \ell, \ell_1, \ell_2 \in L) [(\ell \perp_r \ell_1 \wedge \ell_1 \parallel \ell_2) \rightarrow \ell \perp_r \ell_2]$ , p.1076.

**L<sub>10</sub>**  $\perp_r$  is closed under taking limits, p.1077.

**lopag<sup>+</sup>**  $\stackrel{\text{def}}{=} \text{lopag} + \mathbf{L}_3 + \mathbf{L}_4 + \mathbf{L}_5 + \mathbf{L}_6 + \mathbf{L}_7 + \mathbf{L}_8 + \mathbf{L}_9 + \mathbf{L}_{10}$ , p.1081

**(11) “Speed of light free” axiom systems for relativity**  
**(axioms and axiom systems used in Chapter 5)**

**Relnoph<sub>0</sub>**  $\stackrel{\text{def}}{=} (\mathbf{Ax1-Ax4})^{1304} + \mathbf{Ax6} + \mathbf{Ax}\square\mathbf{1} + \mathbf{Ax}\triangle\mathbf{1} + \mathbf{Ax}(\sqrt{\phantom{x}}) + \mathbf{Ax}(\mathit{Triv}) + \mathbf{Ax}(\parallel)$ , p.705.

**Ax(5nop)**  $\forall m, k (\forall \lambda \in {}^+F)[\lambda < v_m(k) \Rightarrow \exists k' (v_m(k') = \lambda)]$ , p.706.

The intuitive idea of **Ax(5nop)** is that if a certain speed is realized by some observer then the *smaller speeds* are also realized by some observers.

**Relnoph**  $\stackrel{\text{def}}{=} \mathbf{Relnoph}_0 + \mathbf{Ax}(5\text{nop})$ , p.707.

**Ax(group<sup>+</sup>)**  $(\forall m, k, m', k' \in \mathit{Obs})(\exists k'' \in \mathit{Obs})f_{mk} \circ f_{m'k'} = f_{mk''}$ , p.410.

**Ax(syt)\***  $f_{mk}(\bar{0}) = \bar{0} \Rightarrow f_{mk}(1_t)_t = f_{km}(1_t)_t$ , p.721.

**Ax(syx)\***  $(m, k \text{ are in pre-standard configuration})^{1305} \Rightarrow |f_{mk}(1_x)_x| = |f_{km}(1_x)_x|$ , p.725.

(\*1)–(\*3) below are also potential axioms<sup>1306</sup> (or principles) connected with **Relnoph**.

(\*1) The sum of finitely many small positive velocities is not infinite, formally: Let  $j \in \omega$ . Let  $m_0, \dots, m_j \in \mathit{Obs}$ . Assume  $m_i, m_{i+1}$  are in strict standard configuration<sup>1307</sup> and  $v_{m_i}(m_{i+1}) < 1$ , for all  $i < j$ . Then  $v_{m_0}(m_j) \neq \infty$ , p.710.

(\*2) The sum of finitely many small positive velocities is nonnegative, formally: Let  $j \in \omega$ . Let  $m_0, \dots, m_j \in \mathit{Obs}$ . Assume  $m_i, m_{i+1}$  are in strict standard configuration and  $v_{m_i}(m_{i+1}) < 1$ , for all  $i < j$ . Then  $m_0$  sees  $m_j$  moving forwards in direction  $1_x$ , p.710.

<sup>1304</sup> **Ax1, Ax2, Ax3, Ax4.**

<sup>1305</sup> Cf. footnote 1300 on p.1260 for the notion of a pre-standard configuration.

<sup>1306</sup> (\*1)–(\*3) are schemas of formulas.

<sup>1307</sup> Assume  $m, k \in \mathit{Obs}$ . Then  $m$  and  $k$  are defined to be in *strict standard configuration* if they are in standard configuration,  $m$  sees  $k$  moving forwards in direction  $1_x$ ,  $k$  sees  $m$  moving backwards in direction  $1_x$ , further  $[v_m(k) = 0 \Rightarrow f_{km}(1_t)_t \cdot f_{km}(1_x)_x > 0]$  and  $[v_m(k) = \infty \Rightarrow (f_{km}(1_t)_x > 0 \ \& \ f_{km}(1_x)_t < 0)]$ . Cf. Def.5.0.42 (p.709) and Remark 5.0.43 (p.709).

(\*3) The sum of finitely many small positive velocities is nonzero, formally: Let  $0 < j \in \omega$ . Let  $m_0, \dots, m_j \in \text{Obs}$ . Assume  $m_i, m_{i+1}$  are in strict standard configuration and  $0 < v_{m_i}(m_{i+1}) < 1$ , for all  $i < j$ . Then  $v_{m_0}(m_j) \neq 0$ , p.710.

“**Flxspecrel**”, the notation  $\mathfrak{M} \models$  “**Flxspecrel**” was introduced on p.708.

**Ax(natu)**  $(*1) \vee (*2) \vee (*3)$ ,<sup>1308</sup> p.752

**Ax(natu)<sup>+</sup>**  $(\forall m, k, k' \in \text{Obs}) \sqrt{v_m(k')} \leq \sqrt{v_m(k)} + \sqrt{v_k(k')}$ , p.753.

**Ax( $\exists$ body)**  $\forall m, k (\forall \ell \in \text{Eucl}) [(v_m(k) > 0 \wedge f_{mk}[\ell] = \ell) \Rightarrow (\exists b \in B) tr_m(b) = \ell]$ , p.757.

**Ax(5nop)<sup>-</sup>**  $\forall m (\exists c \in {}^+F) (\forall \lambda \in {}^+F) [\lambda < c \Rightarrow (\exists k) v_m(k) = \lambda]$ , p.761.

**Relnoph<sup>-</sup>** is obtained from **Relnoph** by replacing **Ax(5nop)** with **Ax(5nop)<sup>-</sup>**, p.761.

**Bax<sub>nobs</sub><sup>-</sup>**  $\stackrel{\text{def}}{=} \text{Bax}^- \setminus \{\text{Ax5Obs}\} + \text{Ax}(5\text{nop})^-$ , p.762.

**Relnoph<sup>--</sup>**  $\stackrel{\text{def}}{=} \text{Relnoph}^- \setminus \{\text{Ax}\Delta 1\} + \text{Ax}(\text{symm})$ , p.764.

egyezetetni  
§4.3-ben  
definícióval

levő

## (12) The cone-smooth versions of our theories

Assume **Ax1**, **Ax2**, **Ax3<sub>0</sub>**, **AxP1**. Let  $m \in \text{Obs}$ . Then

$$c_m : {}^nF \times \text{directions} \xrightarrow{\circ} F \cup \{\infty\}$$

is a partial function such that  $c_m(p, d)$  is defined iff  $m$  sees a photon at point  $p$  moving forwards in direction  $d$ , and  $c_m(p, d)$  is the speed of this photon,<sup>1309</sup> cf. pp. 473, 535. Further, for any  $m \in \text{Obs}$  and  $d \in \text{directions}$

$$c_m(d) \stackrel{\text{def}}{=} c_m(\bar{0}, d),$$

<sup>1308</sup>**Ax(natu)** is a schema of formulas.

<sup>1309</sup>There is only one such speed because of **AxP1**.

cf. p.488. Hence,  $c_m : \text{directions} \xrightarrow{\circ} F \cup \{\infty\}$  is a partial function. (**Ax(consm)** below will imply that it is not partial. The notion of strongly continuous functions, partial derivatives etc. used in **Ax(consm)** below are formulated in our frame language on pp. 536, 518.) Now,

**Ax(consm)**  $\stackrel{\text{def}}{=} \mathbf{Ax}(\mathbf{cnsm}_0) + \mathbf{Ax}(\mathbf{cnsm}_1) + \mathbf{Ax}(\mathbf{cnsm}_2)$  (cf. p.518), where

**Ax(cnsm<sub>0</sub>)**  $c_m$  is a strongly continuous function defined on directions, p.519.

**Ax(cnsm<sub>1</sub>)** For all  $0 < i < n$ , the partial derivative  $(\partial_i c_m) : \text{directions} \rightarrow F$  is everywhere defined on the domain directions, p.519.

**Ax(cnsm<sub>2</sub>)** For all  $0 < i < n$ ,  $\partial_i c_m$  is strongly continuous on the domain directions, p.519.

Let  $Th$  be one of our theories such that  $Th \models \{ \mathbf{Ax1}, \mathbf{Ax2}, \mathbf{Ax3}_0, \mathbf{AxP1} \}$ . Then

$\boxed{Th_\partial} \stackrel{\text{def}}{=} Th + \mathbf{Ax}(\mathbf{consm})$  is called the cone-smooth version of the theory  $Th$ , cf. p.521.

### (13) Axioms and axiom systems not listed in this list

**Ax<sub>OF</sub>**, p.30.

**Ax<sub>G</sub>**, p.31. (**Ax<sub>OF</sub>** and **Ax<sub>G</sub>** are assumed throughout the present work.)

**Ax1'**, p.45.

**AxE<sub>1</sub>**, p.223.

**AxE<sub>2</sub>**, p.223.

**Ax2<sub>1</sub>**, p.223.

**Ax5<sub>1</sub>**, p.223.

$\boxed{\text{Relphax}}$ , p.223.

**Ax□1'**, p.354.

**Ax△2\***, p.359.

**Ax△3**, p.406.

**Ax(isotropy)**, p.399.

**Ax(isotropy')**, p.400.

**Ax(isotropy<sup>-</sup>)**, p.401.

**Ax(homogeneity)**, p.405.

**Ax(group<sup>-</sup>)**, p.410.

**Ax(group)**, p.410.

$\boxed{\text{Flxspecrel}^+}$ , p.425.

homogeneity

$\mathbf{Ax}2^n$ , p.435.  
 $\mathbf{Ax}5^n$ , p.435.  
 $\mathbf{Ax}E^n$ , p.435.  
 $\boxed{\mathbf{NewtK}^n}$ , p.435.  
 $\mathbf{Ax}5^c$ , p.429.  
 $\mathbf{Ax}E_0^c$ , p.429.  
 $\mathbf{Ax}5^f$ , p.431.  
 $\mathbf{Ax}E_0^f$ , p.431.  
 $\mathbf{Ax}(E_{ess})$ , p.424.  
 $\mathbf{Ax}P1^-$ , p.529.  
 $\mathbf{Ax}5_{Ph}^-$ , p.530.  
 $\mathbf{Ax}5_{Obs}^-$ , p.530.  
 $\boxed{\mathbf{Bax}^{--}}$ , p.531  
 $\mathbf{Ax}P1_1^-$ , p.532.  
 $\mathbf{Ax}P1_2^-$ , p.532.  
 $\mathbf{Ax}P1_3^-$ , p.534.  
 $\boxed{\mathbf{Bax}_1^{--}}$ , p.533.  
 $\boxed{\mathbf{Bax}_2^{--}}$ , p.533.  
 $\boxed{\mathbf{Bax}_3^{--}}$ , p.534.  
 $\boxed{\mathbf{Bax}_+^{--}}$ , p.538.  
 $\mathbf{Ax}(i)$ , p.540.  
 $\mathbf{Ax}(ii)$ , p.540.  
 $\mathbf{Ax}(ii)^+$ , p.542.  
 $\boxed{\mathbf{Bax}_{++}^{--}}$ , p.544.  
 $\boxed{\mathbf{Bax}(P1)}$ , p.544.  
 $\mathbf{R}(\mathbf{Ax}E)^-$ , p.557.  
 $\mathbf{R}(\mathbf{Ax}E_{02})^-$ , p.559.  
 $\mathbf{R}(\mathbf{Ax}E_{00})^-$ , p.559.  
 $\boxed{\mathbf{Reich}_0(Th)^-}$ , p.563.  
 $\mathbf{R}_k(\mathbf{Ax}E)$ , p.574.  
 $\boxed{\mathbf{Reich}'(Th)}$ , p.580.  
 $\mathbf{Ax}R^-$ , p.584.  
 $\mathbf{Ax}R^{1/2}$ , p.584.  
 $\mathbf{Ax}R^+$ , p.584.  
 $\mathbf{Ax}R^{--}$ , p.585.

**Ax(sy)**, p.628.  
**Ax(sy<sub>0</sub>)**, p.629.  
**Ax(5nop)<sup>-+</sup>**, p.763.  
**Ax(cont)**, p.766.  
**Ax(cont)<sup>+</sup>**, p.767.  
**Ax(∃ ↑)**, p.767.  
**Ax(fun)**, p.768.  
**Det**, p.992.  
**det**, p.992.  
**Ax(mild)**, p.1067.  
**G<sub>1</sub>**, p.1172.  
**G<sub>2</sub>**, p.1172.  
**G<sub>3</sub>**, p.1172.  
**G<sub>4</sub>**, p.1172.  
**G<sub>5</sub>**, p.1173.  
**busg**, p.1172.

## Credits

Figures 258, 261, 333 are created from works by M. C. Escher, which appeared in D. R. Hofstadter: “Gödel, Escher, Bach”, Hungarian translation, Typotex Kiadó, 1998. Figure 355 (representing Gödel’s rotating universe) and Figure 290 are created from figures in S. W. Hawking and G. F. R. Ellis [126]. Figure 281 (view from near a black hole) is created from a figure in K. S. Thorne [258].

## Index

- $(\perp_0)_\mu$ , 878  
 $(\perp_0)_m$ , 882  
 $(\exists !x)\psi(x)$ , 947  
 $(x, y)$ , interval, 1179  
 $+_{oe}$ , 1046  
 $< \infty$ , 477  
 $\models$ , 193  
 $R \circ S$ , 26  
 $[oe]$ , 1045  
 $[x]_I$ , 449  
 $\mathfrak{A} \xrightarrow{\gamma} \mathfrak{B}$ , 1008  
 ${}^n\mathbf{F}$ , 42, 43  
 ${}^n\mathbf{F}_1$ , 42  
 ${}^n\mathbf{F}_2$ , 42  
 $\|-\|$ , 1096  
 $\|p\|$ , 189  
 $\bigcup K$ , 869  
 $\perp$ , 785, 790  
 $\perp_r^\omega$ , 821  
 $\perp_r^i$ , 822  
 $\perp_0$ , 791  
 $\perp_0$ -version of the Minkowskian geometry, 878  
 $\perp_\mu$ , 860  
 $\perp_e$ , 172  
 $\perp_m$ , 881  
 $\perp_r$ , 792  
 $\perp_r'$ , 810  
 $\perp_r''$ , 810, 811  
 $\perp_r'''$ , 822  
 $\cdot_{oe}$ , 1046  
 $\cong$ , relation of isomorphism between structures, 790  
 $\cong_I$ , 447  
 $\equiv_\Delta^*$ , 970  
 $\equiv_\Delta^w$ , weak definitional equivalence, 986  
 $\equiv_\Delta$ , 970  
 $\longrightarrow$ , 983  
 $\ell \perp_e \ell'$ , 172  
 $\ell \sim \ell'$ , 894  
 $\ell_1 \parallel \ell_2$  when  $\ell_1, \ell_2 \in \text{lines}$ , 1039  
 $\tau_p$ , 152  
 $\equiv^S$ , 799  
 $\equiv^T$ , 799  
 $\equiv^{Ph}$ , 799  
 $\equiv_{ee}$ , 272  
 $\xrightarrow{\gamma}$ , 983, 1008  
 $\xrightarrow{\gamma}$ , 983  
 $\infty$ , 46, 477  
 $\iota_x$ , 333  
 $\langle a, b \rangle \parallel \langle c, d \rangle$ , 1042  
 $\lambda < \infty$ , 477  
 $\lambda \cdot -$ , 1096  
 $\overset{\circ}{\rightarrow}$ , 192  
 $\langle \text{expr}(x) : x \in D \rangle$ , 27  
 $\leq_{\mathcal{M}o}$ ,  $\leq_{\mathcal{G}o}$ ,  $\leq_{\mathcal{M}}$ ,  $\leq_{\mathcal{G}}$ , 1082  
 $\leq_{oe}$ , 1046  
 $\longleftarrow$ , 1008  
 $\models$ , 28, 35, 161  
 $A \xrightarrow{f} B$ , 26  
 $\neq$ , see footnote 207 on p, 197  
 $\models^{\text{OFG}}$ , 35  
 $\omega$ , 26  
 $\ominus^m v$ , 420  
 $\overline{pq}$ , 68  
 $\overline{pq}$ , 164  
 $\|$ , 164, 1042  
 $\|_{\mathfrak{G}}$ , 790, 799  
 $\partial_i f$ , 518  
 $\prec$ , 789  
 $\prec_\mu$ , 860  
 $\prec_m$ , 881

$\succ$ , 1171  
 $\sqcup$ , 1169  
 $\psi(x/\tau)$ , 945  
 $\psi(y)$ , 945  
 $\xrightarrow{\circ}$ ,  $f : A \xrightarrow{\circ} B$ , 517  
 $\mathfrak{F}^\infty$ , 535  
 $\sim$ , 818, 894  
 $\cong$ , 1081  
 $\bigcup_{i \in I} \mathfrak{G}_i$ , 871, 874  
 $\bigcup K$ , 869, 870  
 $\gamma_\mu^r$ , 1165  
 $\gamma^r$ , 1156  
 $\tau(-)$ , 1096  
 $\mathbf{f}_{mk}^-$ , 655  
 $\sigma_S$ , 359  
 $\sigma_{\bar{t}}$ , 306  
 $\sigma_\ell$ , 173, 367  
 $\vec{\ell}_{o,e}$ , 891  
 $\tilde{\varphi}$ , 154  
 $\subseteq_w$ , 1008  
 $\mathbf{f}_{mk} \equiv_v \mathbf{f}_{m'k'}$ , 421  
 $d \parallel d_1$ , when  $d, d_1 \in {}^{n-1}F$ , 471  
 $f \circ g$ , 26  
 $m \xrightarrow{\circ} b$ , 192  
 $v \oplus^m w$ , 420  
 $+F$ , 189  
 ${}^nH$ , 26  
 ${}^nF$ , 32, 43  
 $({}^nF) {}^nF$ , 135  
 $|a|$ , 30  
 $|p|$ , 100  
 $1$ , 30  
 $S$ , 54  
 $\mathfrak{M}_{\mathfrak{F}}^M$ , 331  
 $\perp$ , 792  
 $\perp_e$ , 172  
 $\perp_r$ , 790  
 $\ell_1 \parallel \ell_2$ , 164  
 $\equiv_v$ , 421  
 $f'$ , derivative of  $f$ , 517  
 $g(e, e_1)$ , 797  
 $p \perp_e q$ , 172  
1/2-simultaneity, 582  
1/2-simultaneous, 584  
2-dimensional plane in a relativistic geometry, see footnote 798 on p, 857  
 $1_i$ , 69  
1/2-simultaneous, 582  
 $1_t, 1_x, 1_y, 1_z$ , 69  
 $\forall\text{GlobFTL}$ , 686  
 $\mathbf{Ax}(\mathbf{Bw})^{\text{par}}$ , 664  
 $\mathbf{Ax}(\text{Frame})$ , 658  
 $\mathbf{Ax}(\text{clock-conn})$ , 660  
 $\mathbf{Ax}(\text{continuity})$ , 661  
 $\mathbf{Ax}(\text{eqspace})$ , 406  
 $\mathbf{Ax}(\text{mut})$ , 661  
 $\mathbf{Ax}(\text{pcoll})$ , 661  
 $\mathbf{Ax}(\text{photon})$ , 660  
 $\mathbf{Ax}(\text{simult})$ , 660  
 $\mathbf{Ax}(\text{star})$ , 659  
 $\mathbf{Ax}(\text{syBw})^{\text{par}}$ , 664  
 $(\text{abspc})$ , 438  
 $(\text{abstime})$ , 437  
 $\mathbf{A}_1$ , 1038  
 $\text{Afr} = \text{Afr}(n, \mathfrak{F})$ , 152  
 $\mathbf{ag}$ , 1040  
 $\mathbf{A}_3$ , 1039  
 $\mathbf{A}_2$ , 1039  
 $\forall\text{LocFTL}$ , 686  
 $\mathbf{A}_4$ , 1039  
 $\text{ang}^2(\ell)$ , 46  
 $\text{ang}^2(d, d')$ , 538  
 $\mathbf{A}_0$ , 1038  
 $\mathbf{Asim}(K)$ , 570  
 $\mathbf{Asim}(Th)$ , 570  
 $a$ -simultaneity, 444

“async”, 652  
**(asynch)**, 436  
 $Aut(\mathfrak{A})$ , 154, 161  
 $Aut^F(\mathfrak{G}_m)$ , 915  
 $Aut(\mathfrak{F})$ , 154  
**Ax1**, 45  
**Ax1'**, 45  
**Ax4<sup>par</sup>**, 658  
**Ax2**, 48  
**Ax3**, 48  
**Ax3<sub>0</sub><sup>par</sup>**, 658  
**Ax4**, 48  
**Ax5**, 50  
**Ax(5<sup>Obs</sup>)<sup>par</sup>**, 662  
**Ax(5<sup>Ph</sup>)<sup>par</sup>**, 662  
**Ax5<sup>par</sup><sub>Obs</sub>**, 658  
**Ax5<sup>par</sup>**, 661  
**Ax5<sup>par</sup><sub>Ph</sub>**, 659  
**Ax6**, 50  
**Ax6<sub>00</sub><sup>par</sup>**, 702  
**Ax6<sub>01</sub><sup>par</sup>**, 659  
**Ax7**, 264  
**Ax(Bw)**, 1028  
**Ax( $\omega$ )<sup>00</sup>**, 844  
**Ax(cnsm<sub>0</sub>)**, 519  
**Ax(cnsm<sub>1</sub>)**, 519  
**Ax(cnsm<sub>2</sub>)**, 519  
**Ax( $\omega$ )<sup>0</sup>**, 844  
**Ax(consm)**, 518  
**Ax(cont)**, 766  
**Ax(cont)<sup>+</sup>**, 767  
**Ax(diswind)**, 812  
**AxE**, 51  
**Ax( $\exists$ body)**, 757  
**AxE<sub>1</sub>**, 223  
**AxE<sup>n</sup>**, 435  
**AxE<sub>2</sub>**, 223  
**AxE<sub>0</sub>**, 191  
**AxE<sub>0</sub><sup>c</sup>**, 429  
**AxE<sub>01</sub>**, 218  
**AxE<sub>0</sub><sup>f</sup>**, 431  
**AxE<sub>02</sub>**, 427  
**AxE<sub>00</sub>**, 218  
**Ax(eqm)**, 397  
**Ax(eqm)**, 796  
**Ax(eqspace)**, 136  
**Ax(eqtime)**, 127  
**Ax(eqtime)<sup>par</sup>**, 697  
**Ax( $E_{ess}$ )**, 424  
**Ax( $\exists$ TWP)**, 461  
**Ax(ext)**, 298  
**Ax(fun)**, 768  
**Ax<sub>G</sub>**, 31  
**Ax(group)**, 410  
**Ax(group<sup>-</sup>)**, 410  
**Ax(group<sup>+</sup>)**, 410  
**Ax3<sub>0</sub>**, 191  
**Ax6<sub>01</sub>**, 190  
**Ax6<sub>00</sub>**, 190  
**Ax $\Delta$ 1**, 351  
**Ax $\Delta$ 2**, 351  
**Ax $\Delta$ 2\***, 359  
**Ax $\Delta$ 3**, 406  
**Ax(homogeneity)**, 405  
**Ax(i)**, 540  
**Ax(ii)**, 540  
**Ax(ii)<sup>+</sup>**, 542  
**Ax(isotropy)**, 399  
**Ax(isotropy<sup>?</sup>)**, 401  
**Ax(isotropy<sup>-</sup>)**, 401  
**Ax( $\uparrow\uparrow$ )**, 426  
**Ax( $\uparrow\uparrow_0$ )**, 840  
**Ax( $\uparrow\uparrow_{00}$ )**, 840  
**Ax2<sub>1</sub>**, 223  
**Ax2<sup>n</sup>**, 435  
**Ax(mild)**, 1067  
**Ax(natu)**, 752  
**Ax(natu)<sup>+</sup>**, 753

$\text{Ax}(\text{Triv})$ , 135, 297, 601  
 $\text{Ax}(\text{Triv}_t)$ , 135, 297, 601  
 $\text{Ax}(\exists \uparrow)$ , 767  
 $\text{Ax}\square 1$ , 350, 426  
 $\text{Ax}\square 2$ , 350  
 $\text{Ax}\square 1'$ , 354  
 $\text{Ax}(\uparrow)$ , 296  
 $\text{Ax}_{\text{OF}}$ , 30  
 $\text{Ax}(\omega)$ , 351  
 $\text{Ax}(\omega^-)$ , 351  
 $\text{Ax}(\omega)^\#$ , 844  
 $\text{Ax}(\omega)^\#\#$ , 844  
 $\text{Ax}5^c$ , 429  
 $\text{Ax}5_1$ , 223  
 $\text{Ax}5^f$ , 431  
 $\text{Ax}5^n$ , 435  
 $\text{Ax}(5\text{nop})$ , 706  
 $\text{Ax}(5\text{nop})^-$ , 761  
 $\text{Ax}(5\text{nop})^{-+}$ , 763  
 $\text{Ax}5^{\text{Obs}}$ , 218  
 $\text{Ax}5_{\text{Obs}}$ , 477  
 $\text{Ax}5_{\text{Obs}}^-$ , 530  
 $\text{Ax}5_{\text{Obs}}^{--}$ , 481  
 $\text{Ax}5^+$ , 297  
 $\text{Ax}5^{\text{Ph}}$ , 219  
 $\text{Ax}5_{\text{Ph}}$ , 477  
 $\text{Ax}5_{\text{Ph}}^-$ , 530  
 $\text{AxP1}$ , 472  
 $\text{Ax}(\parallel)$ , 136, 457  
 $\text{Ax}(\parallel)^-$ , 828  
 $\text{Ax}(\infty ph)$ , 1028  
 $\text{Ax}(Ph)$ , 1073  
 $\text{AxP1}^-$ , 529  
 $\text{AxP1}_1^-$ , 531  
 $\text{AxP1}_3^-$ , 533  
 $\text{AxP1}_2^-$ , 532  
 $\text{Ax}(\text{rc})$ , 301  
 $\text{AxR}^{1/2}$ , 584  
 $\text{AxR}^-$ , 584

$\text{AxR}^{--}$ , 585  
 $\text{AxR}^+$ , 584  
 $\text{Ax}(\text{speedtime})$ , 137  
 $\text{Ax}(\text{sy})$ , 628  
 $\text{Ax}(\text{symm})^\dagger$ , 426  
 $\text{Ax}(\text{symm})$ , 127  
 $\text{Ax}(\text{symm}_0)^{\text{par}}$ , 696  
 $\text{Ax}(\text{symm}_0)^{\text{par}+}$ , 696  
 $\text{Ax}(\text{symm})^\dagger$ , 151  
 $\text{Ax}(\text{symm}_0)$ , 124  
 $\text{Ax}(\text{sy}_0)$ , 629  
 $\text{Ax}(\text{syt})$ , 386, 457  
 $\text{Ax}(\text{syt}_0)^{\text{par}}$ , 694  
 $\text{Ax}(\text{syt})^*$ , 721  
 $\text{Ax}(\text{syt}_0)$ , 134, 457  
 $\text{Ax}(\text{syx})^*$ , 725  
 $\text{Ax}\heartsuit$ , 296  
 $\text{Ax}(\sqrt{\quad})$ , 91  
 $\text{Ax}(\sqrt{\quad})$ , 481  
 $\text{Ax}(\text{Triv}_t)^-$ , 812  
 $\text{Ax}(\text{TwP})$ , 140  
 $B$ , 29  
 $B$ , 29  
 $B_1$ , 1043  
 $B_2$ , 1043  
 $B_3$ , 1043  
 $BA$ , 1017, 1018  
 $BaCo$ , 298  
 $Basax$ , 51  
 $Basax$  geometry, 799  
 $Bax$ , 219  
 $Bax_2^{--}$ , 533  
 $Bax_1^{--}$ , 533  
 $Bax_3^{--}$ , 534  
 $Bax^-$ , 479  
 $Bax_\theta^-$ , 521  
 $Bax^{--}$ , 531  
 $Bax_+^{--}$ , 538  
 $Bax_{++}^{--}$ , 544

**Bax**<sub>nobs</sub><sup>-</sup>, 762  
**Bax(P1)**, 544  
 Betw( $p, r, q$ ), 790  
 Betw( $p, r, q$ ), 492  
 Bw, 790  
 Bw <sub>$\mu$</sub> , 860  
 Bw <sub>$m$</sub> , 881  
 c, 428  
 C, 492  
 Ch( $H$ ), convex hull of  $H \subseteq Mn$ , 838  
 (clock), 436  
 (clock)<sup>-</sup>, 438  
 c <sub>$m$</sub> , c <sub>$m$</sub> ( $d$ ), 488  
 C <sub>$m$</sub> , 505  
 c <sub>$m$</sub> , 535  
 c <sub>$m$</sub> ( $p, d$ ), 535  
 c <sub>$m$</sub> <sup>f</sup>( $p, d$ ), 542  
 c <sub>$m$</sub> ( $p, d$ ), 473  
 c <sub>$m$</sub> ( $p, d$ ) <  $\infty$ , 491  
 C <sub>$m,t$</sub> , 505  
 Co <sub>$\langle o, e_0, \dots, e_{n-1} \rangle$</sub> , 1051  
 code, 966, 967  
 coll( $a, b, c$ ), 818  
 coll<sub>F</sub>, coll<sub>D</sub>, 1040  
 Col, 991  
 Col<sup>T</sup>, Col<sup>Ph</sup>, Col<sup>S</sup>, 998  
**Compl**, 298  
 Cone <sub>$m,p$</sub> , 473  
**Det**, 992  
**det**, 992  
 directions, 470  
 Dom( $R$ ), 26  
 (E1), 635  
 E <sub>$m$</sub> <sup>1/2</sup>, 584  
 (E1)<sup>par</sup>, 698  
 (E2), 635  
 (E2)<sup>par</sup>, 698  
 (E3), 636  
 (E3)<sup>par</sup>, 699  
 (E3)<sup>par $\perp$</sup> , 699  
 (E3)<sup>par $\parallel$</sup> , 699  
 (E4), 636  
 (E5), 638  
 (E5)<sup>+</sup>, 641  
 (E5)<sup>+ $\ddagger$</sup> , 649  
 (E6), 641  
 (E6)<sup>par</sup>, 700  
 (E7), 641  
 (E7)<sup>par</sup>, 700  
 EC <sub>$\Delta$</sub> , 451  
 EC <sub>$\Delta$</sub> , 452  
 EC <sub>$\Delta,0$</sub> , 451  
 $\exists$ GlobFTL, 686  
 (Ei)(or), 640  
 (Ei)(fast), 640  
 $\exists$ LocFTL, 686  
 eq, 795  
 $\langle a, b \rangle$  eq  $\langle c, d \rangle$ , 795  
 eq <sub>$m$</sub> , 881  
 eq<sub>0</sub>, 793  
 eq<sub>0</sub>-witness, 899  
 eq <sub>$i$</sub> , 795  
 eq <sub>$\mu$</sub> , 860  
 eq<sub>0</sub><sup>S</sup>, 902  
 eq<sub>0</sub><sup>S</sup>-witness, 902  
 Eucl = Eucl( $n, \mathfrak{F}$ ), 45  
 Euclgeom<sup>0</sup>( $n, \mathfrak{F}$ ), Euclgeom<sup>0</sup>( $\mathfrak{F}$ ), 1129  
 Euclgeom( $n, \mathfrak{F}$ ), Euclgeom( $\mathfrak{F}$ ), 1129  
 Eudist, 609  
 Exp = Exp( $n, \mathfrak{F}$ ), 153  
 expr(-), 1096  
 F, 30, 43  
 F <sup>$\infty$</sup> , 535  
 $f : A \longrightarrow B$ , 26, 1008  
 F<sub>0</sub> =  $\langle F; 0, +, \leq \rangle$ , 1151  
 F<sub>1</sub> =  $\langle F; 0, 1, +, \leq \rangle$ , 788  
 $\mathfrak{F}$ , 30, 43  
 F, 30, 43

$f^2 := f \circ f$ , if  $f$  is a function, 1011  
 $\mathbf{F}^m$ , 30  
 $f[\mathcal{A}]$ , 1085  
 “fat”, 652  
 $\mathfrak{F}$ -categorical, 299  
**Flxbasax**, 428  
**Flxspecrel**, 428  
**Flxspecrel**<sup>+</sup>, 425  
 FM, 35, 1006  
 $Fm$ , 28  
 $\mathfrak{F}^m$ , 30  
 $\widehat{f}_{mk}$ , 56  
 $f_{mk}$ , 915  
 $f_{mk}(p)$ , 61  
 $Fm(\mathbf{K})$ , 962  
 $Fm(Th)$ , 1020  
 $\mathfrak{F}_{oe}$ , 1046  
 $F_{oe}$ , 1046  
 $f \upharpoonright C$ , 27  
 $f(x)$ , 26  
 $f[X]$ , 27  
 $G$ , 30  
 $G$ , 29  
 $g$ , 797  
 $g(e, e_1)$ , 796  
 $g$ , pseudo-metric, 796  
 $Ge(Th)$ , 798  
 $Ge^0(Th)$ , 1070  
 $Ge^{\perp_0}(Th)$ , 871  
 $\mathbf{G}_1$ , 1171  
 $Ge^{inc}(Th)$ , 1174  
 $Ge^i(Th)$ , 1125  
 GEO, 1070  
**geod**, 52  
 $Geom(Th)$ , 798  
 $Ge^{\prec}(Th)$ , 1174  
 $Ge^R(Th)$ , 1169  
 $Ge_{Ta}$ , 993  
 $Ge(Th)$ , 1085  
 $Ge'(Th)$ , 847  
 $Ge''(Th)$ , 847  
 $Ge_{We}$ , 993  
 $\mathcal{G}$ , 1007  
 $\mathcal{G}_0 \dot{\cup} \mathcal{G}_1$ , 871  
 $\mathcal{G}_m$ , 880  
 $\mathcal{G}_m^{\perp_0}$ , 881  
 $\mathcal{G} : \text{Mod}(Th) \longrightarrow Ge(Th)$ , 1008  
 $\mathcal{G}_m$ , 787  
 $\mathcal{G}_m^*$ , 1111  
 $\mathcal{G}_m^0$ , 1070, 1124  
 $\mathcal{G}'_m$ , 846  
 $\mathcal{G}_m^1$ , 1124  
 $\mathcal{G}''_m$ , 846  
 $\mathcal{G}_m^2$ , 1124  
 $\mathcal{G}_m^3$ , 1124  
 $\mathcal{G}_m^4$ , 1124  
 $\mathcal{G}_m^5$ , 1125  
 $\mathcal{G}_m^6$ , 1129  
 $\mathcal{G}_m^7$ , 1129  
 $\mathcal{G}_m^{\perp_0}$ , 871  
 $\mathcal{G}_m^{\equiv}$ , 799  
 $\mathcal{G}_m^R$ , 799  
 $\mathcal{G}$ -parallel, 790  
 $\mathcal{G} \upharpoonright N$ , 882  
 $\mathcal{G} \upharpoonright^* N$ , 1130  
 $\mathcal{G} \upharpoonright^+ N$ , 882  
 $\mathbf{G}_3$ , 1171  
 $\mathcal{G}_m^{inc}$ , 1174  
 $\mathbf{G}_2$ , 1171  
 GM, 269  
 $g_m$ , 881  
 $\mathbf{G}_m$ , 1209  
 $\mathbf{G}_m$ , 787  
 $\mathbf{G}_m$ , 787  
 $(\mathcal{G}_m^R)^{\equiv}$ , 800  
 $g_\mu$ , 860  
 $g_\mu^2$ , 152  
 $g_\mu^2(p, q)$ , 152

**G**<sub>4</sub>, 1171  
*G*<sub>o</sub>, *M*<sub>o</sub>, 1072  
**G**<sub>5</sub>, 1172  
*g*<sup>↖</sup>, 1169  
*g*<sup>R</sup>(*e*, *e*<sub>1</sub>), 800  
**G**<sub>Ta</sub>, 991  
*GT*<sub>ℳ</sub>, 923  
*GT*<sub>ℳ</sub><sup>*i*</sup>,  $-2 \leq i \leq 2$ , 925  
**G**<sub>We</sub>, 991  
*g*( $-, y, z$ ), *g*( $x, -, z$ ), *g*( $x, y, -$ ), 518,  
1096  
**I**, 785  
*Ib*, 30  
*Id*, 27  
*Id*<sub>A</sub>, 27  
*i*-dimensional plane in  $\langle Mn; Bw \rangle$ , 1039  
**IK**, 785  
*inf*, 451  
*int*(*p*, *q*), 509  
*j*-dimensional plane, 164  
**K** is definable implicitly over **L**, 936  
*k* sees *p*, 482  
**K**  $\uparrow$  *Voc*, 933  
*k*-way speed of light, 573  
**L**, 789  
**L**<sub>1</sub>, 1071  
*LG*<sub>ℳ</sub>, 408  
*LG*(*Th*, **n**), 409  
**L**<sub>3</sub>, 1076  
**L**<sub>6</sub>, 1076  
**L**<sub>7</sub>, 1076  
*LightCone*( $\bar{0}$ ), 207  
*LightCone*(*p*), 207  
*Linb* = *Linb*(*n*,  $\mathfrak{F}$ ), 152  
*lines*, defined sort of  $\langle Mn; Bw \rangle$  or  
 $\langle Mn; coll \rangle$ , 1037  
*Lines* = *L*  
cf. footnote 958 on p, 991  
**L**<sub>2</sub>, 1071  
**L**<sub>9</sub>, 1076  
*L*<sub>m</sub>, 880  
*L*<sub>ℳ</sub>, 790  
*L*<sub>μ</sub>, 859  
*L*<sub>μ</sub><sup>Ph</sup>, 860  
*L*<sub>μ</sub><sup>S</sup>, 860  
*L*<sub>μ</sub><sup>T</sup>, 859  
**L**<sub>4</sub>, 1076  
**L**<sub>8</sub>, 1076  
*Loc*<sub>1</sub>(*Th*), 702  
**lopag**, 1071  
**lopag**<sup>+</sup>, 1081  
*Lor* = *Lor*(*n*,  $\mathfrak{F}$ ), 152  
**L**<sub>5</sub>, 1076  
*L*<sup>Ph</sup>, 788  
*L*<sub>m</sub><sup>Ph</sup>, 880  
*L*<sub>m</sub><sup>R</sup>, 800  
*L*<sup>S</sup>, 788  
*L*<sub>m</sub><sup>S</sup>, 881  
*L*<sup>T</sup>, 788  
**L**<sub>10</sub>, 1077  
*L*<sub>m</sub><sup>T</sup>, 880  
**M**, 28  
*Mod*(*Th*), 1085  
**(meter)**, 436  
**(meter)**<sup>=</sup>, 438  
**(meter)**<sup>≥</sup>, 438  
**(meter)**<sup>≤</sup>, 438  
**M**, **G**, **M**<sub>o</sub>, **G**<sub>o</sub>, 1087  
*min*, see footnote 665 on p, 796  
*Mink*<sup>1<sup>o</sup></sup>( $\mathfrak{F}$ ), 878  
*Mink*<sub>nonE</sub>(*n*,  $\mathfrak{F}$ ), 1160  
*Mink*<sub>nonE</sub>( $\mathfrak{F}$ ), 1160  
**Mink**(*n*), 1159  
**Mink**(*n*, *rc*), 1116  
*Mink*(*n*,  $\mathfrak{F}$ ), 859  
*Mink*( $\mathfrak{F}$ ), 859  
**M**, 1054  
**ℳ'** is obtained from **ℳ** by step (1), 946

$\mathfrak{M}'$  is obtained from  $\mathfrak{M}$  by Step (2.1),  
947  
 $\mathfrak{M}'$  is obtained from  $\mathfrak{M}$  by Step (2.2),  
950  
 $\mathfrak{M} \models \psi[\bar{a}]$ , 945  
 $\mathfrak{M} \cup \mathfrak{N}$ , 869  
 $\text{Mog}(TH)$ , 1085  
 $\mathfrak{M}_{\mathfrak{F}}^Q$ , 325  
 $\mathfrak{M} \upharpoonright \text{Voc}$ , 932  
 $\mathfrak{M}/P$ , 569  
 $\mathfrak{M} \dot{\cup} \mathfrak{N}$ , 868, 869  
 $\mathfrak{M} \subseteq \mathfrak{N}$ , 161  
 $Mn$ , 787  
 $Mn_{\mathfrak{M}}$ , 790  
 $\text{Mod}(Th)$ , 1006  
 $\text{Mod}_{\text{Arch}}(Th)$ , 757  
 $\text{Mod}_{\mathfrak{F}}(\Sigma)$ , 194  
 $\text{Mod}_{\text{OFG}}(Th)$ , 35  
 $\text{Mod}(\Sigma)$ , 35  
 $\text{Mog}(TH)$ , 1071  
 $\text{Mor } \mathbb{C}$ , 1085  
 $\text{MS}(\mathfrak{M}, m)$ , 89  
 $\text{MS}$ , 89  
 $m$ -space like, 566  
 $m \uparrow k$ ,  $m \downarrow k$ , 296  
 $\mathfrak{N} \models \text{“}Th\text{”}$ , 708  
 $n$ , 29, 44  
 $\varepsilon$ -neighborhood, 189  
 $\varepsilon$ -neighborhood of an event in an  
observer-independent (or a rel-  
ativistic) geometry, 797  
**Newbasax**, 191  
**NewtK**, 426  
**NewtK**<sup>-</sup>, 426  
**NewtK**<sup>n</sup>, 435  
“noftl”, 652  
 $N_{pq}$ , 268  
**ObC**, 1085  
*Obs*, 30  
**opag**, 1044  
 $\text{Open} = \text{Open}(n, \mathfrak{F})$ , 190  
**Ordinals**, 791  
 $\mathcal{P}(H)$ , 26  
(P1), 522  
(P2), 522  
(P3), 522  
**Pa**, 1042  
**pag**, 1042  
**Pax**, 482  
**Pax**<sup>+</sup>, 1086  
**Pax**<sup>++</sup>, 1093  
**Pax**<sup>+</sup>, 1029  
**Pax**<sup>++</sup>, 1081  
**P**<sub>1</sub>, 1040  
 $\text{PG}_{\mathfrak{M}}$ , 408  
 $\text{PG}(Th, \mathfrak{n})$ , 408  
*Ph*, 30  
**PhtEucl** = **PhtEucl**( $n, \mathfrak{F}$ ), 58  
 $p_i$ , 42  
 $p_{j_i}$ , 947  
**P**<sub>2</sub>, 1040  
 $\text{Plane}(\ell_1, \ell_2)$ , 820  
 $\text{Plane}(H)$ , 819  
 $\text{Plane}'(H)$ , 820  
 $\text{Plane}(\ell_1, \ell_2)$ , 164  
**Planes**( $n, \mathbf{F}$ ), 532  
**Planes**, 532  
 $\text{Plane}(\bar{t}, \bar{x})$ , 69  
 $\langle \text{Points}; \text{Col} \rangle$ , 991  
 $\langle \text{Points}, \text{Lines}; \in \rangle = \langle Mn, L; \in \rangle$ , 991  
 $\text{Points} = Mn$   
cf. footnote 958 on p, 991  
 $\text{Poi} = \text{Poi}(n, \mathfrak{F})$ , 153  
 $\text{PT}^M = \text{PT}^M(n, \mathfrak{F})$ , 332  
 $\text{PT}^Q = \text{PT}^Q(n, \mathfrak{F})$ , 327  
 $\text{PT}$ , 265  
 $p_t$ , 44  
**Pth**, 541

$p_x$ , 44  
 $p_y$ , 44  
 $p_z$ , 44  
 $Q$ , 29  
 $\mathfrak{R}$ , 26  
 $R$ , 26  
 $R^{-1}$ , 27  
 $\mathbf{R}(\mathbf{AxE})$ , 557  
 $\mathbf{R}(\mathbf{AxE})^-$ , 557  
 $\mathbf{R}(\mathbf{AxE}_0)$ , 557  
 $\mathbf{R}(\mathbf{AxE}_{00})$ , 557  
 $\mathbf{R}(\mathbf{AxE}_{00})^-$ , 559  
 $\mathbf{R}(\mathbf{AxE}_{02})$ , 557  
 $\mathbf{R}(\mathbf{AxE}_{02})^-$ , 559  
 $\mathbf{R}(\mathbf{Ax eqsp})$ , 611  
 $\mathbf{R}(\mathbf{sym})$ , 616  
 $\mathbf{R}(\mathbf{Ax syt}_0)$ , 615  
 $\mathbf{R}_\Delta(E)$ , 574  
 $\mathbf{Reich}(Th)$ , 576  
 $\mathbf{Reich}$ , 563  
 $\mathbf{Reich}'(Th)$ , 580  
 $\mathbf{Reich}_0(Th)$ , 562  
 $\mathbf{Reich}_0(Th)^-$ , 563  
 $\mathbf{Reich}_0(Th)_\partial$ , 562  
 $\mathbf{Relnoph}$ , 707  
 $\mathbf{Relnoph}^-$ , 761  
 $\mathbf{Relnoph}^{--}$ , 764  
 $\mathbf{Relnoph}_0$ , 705  
 $\mathbf{Relphax}$ , 223  
 $rep$ , 966, 967  
 $Rhomb$ , 74  
 $Rhomb^Q = Rhomb^Q(n, \mathfrak{F})$ , 327  
 $Rhomb^M = Rhomb^M(n, \mathfrak{F})$ , 332  
 $\mathbf{R}_k(\mathbf{AxE})$ , 574  
 $Rng(R)$ , 26  
 $R[X]$ , 27  
 $S$ , 208  
 $S(e, \varepsilon)$ , 797  
 $S$ , space-part, 470  
 $S_1$ , 492  
 $S_m^{1/2}$ , 584  
 $S_m^{1/2}$ -simultaneous, 584  
 $S(a)$ , 444  
 $S''(a, b)$ , 839  
 $S'(H)$ , 838  
“shrink”, 652  
simplexes, 838  
 $SL_{\mathfrak{M}}$ , 408  
 $SLor = SLor(n, \mathfrak{F})$ , 153  
“slow”, 652  
 $SlowEucl = SlowEucl(n, \mathfrak{F})$ , 58  
 $SL(Th, \mathbf{n})$ , 409  
 $SM$ , 249  
 $space(p)$ , 470  
 $spacep$ , 438  
 $space$ , 470  
 $S(p, \varepsilon)$ , 189  
 $\mathbf{Specrel}$ , 151  
 $S^R(e, \varepsilon)$ , 800  
 $STL$ , 110, 139, 460  
 $suc$ , 939  
 $sup$ , 451  
 $\bar{t}$ , 43  
 $\bar{t}$ , time-part, time-axis, 470  
 $\mathcal{T}$ , 797  
 $\mathcal{T}'$ , alternative version of the topology  
part  $\mathcal{T}$  of observer-  
independent geometry, 839  
 $\mathcal{T}''$ , alternative version of the topology  
part  $\mathcal{T}$  of observer-  
independent geometry, 839  
 $T_0$ , a subbase for the topology  $\mathcal{T}$ , 797  
 $T'_0$ , a subbase for the topology  $\mathcal{T}'$ , 839  
 $T''_0$ , a subbase for the topology  $\mathcal{T}''$ , 839  
 $T_0^R$ , 800  
 $\bar{t}$ -axis, 43  
 $TH$ , 451  
 $Th^{+-}$ , 828

$\text{Th}(\mathfrak{F})$ , 29  
 $\text{TH}$ , 452  
 $Th$  geometry, 799  
 $\text{TH}_0$ , 451  
 $Th_1 \models "Th"$ , 708  
 $\text{Th}_1 \models \models \text{Th}_2$ , 193  
 $Th_\partial$ , 521  
 $\text{Th}(\mathfrak{F})$ , 301  
 $\text{Th}(\mathfrak{M})$ , 29  
 $Th^\oplus$ , 643  
 $Th$ -potential law (of nature), 1109, 1110  
 $Th$ -simultaneity-stable, 578  
 $time(p)$ , 470  
 $time$ , 470  
 $\mathcal{T}_\mu$ , 861  
 $\mathcal{T}_m$ , 881  
 $\text{T}(Axi)$ , theory generated by the axiom system  $Axi \subseteq Fm$ , 1027, 1111  
 $T_m(d)$ , 555  
 $T_{pq}$ , 268  
 $\mathcal{T}^R$ , 800  
 $Tran = Tran(n, \mathfrak{F})$ , 152  
 $Triv = Triv(n, \mathfrak{F})$ , 135, 247, 601  
 $Triv_0 = Triv_0(n, \mathfrak{F})$ , 247  
 $tr_m(b)$ , 47  
 $Tr$ , 966, 967  
 $Tr_k$ , 967  
 $\bar{t}$ -symmetric, 306  
"twin", 652  
 $(\mathbf{TwP})$ , 38  
 $U_V(\mathfrak{M})$ , 929  
 $Var(U_i)$ , 964  
 $(\mathbf{vel})$ , 437  
 $view_m$ , 615  
 $VL_{\mathfrak{M}}$ , 420  
 $v_m(b)$ , 655  
 $v_m(b)$ , 47  
 $\vec{v}_m(b)$ , 48  
 $Voc \cap Voc'$ , 933  
 $Voc \cup Voc'$ , 933  
 $Voc(\mathfrak{M})$ , 931  
 $Voc(\mathbf{K})$ , 932  
 $W$ , 32  
 $\mathbf{Wax}$ , 1073  
 $\mathbf{Wax}^+$ , 1081  
 $w_m$ , 32  
 $w_m : {}^nF \xrightarrow{\circ} (??)$ , cf. footnote 198 on p, 188, 1210  
 $w_m^-$ , 654  
 $Wtm$ , 60  
 $\bar{x}$ , 43  
 $\bar{x}_i$ , 44  
 $\mathbf{X} = \langle X, \mathcal{O} \rangle, \langle Mn, \mathcal{T} \rangle$ , topological space, 870  
 $\bar{y}$ , 44  
 $\bar{z}$ , 44  
3-sorted first-order language, 27, 29  
0, 30  
 $\bar{0}$ , 42  
 $(*1)$ , 710  
 $(*2)$ , 710  
 $(*3)$ , 710  
(A)–(I), theorem schemas for duality theories, 1012  
absolute space, 438  
absolute time, 437  
abstract class of structures, see footnote 643 on p, 786, 799  
abstract structure, see footnote 643 on p, 786  
abstract/concrete distinction, cf. footnote 643 on p, 786  
additive (geodesic, quasi geodesic), 1183  
additive,  $g \upharpoonright D$  is additive, 1183  
adequate, 607

adequate (a symmetry principle being adequate for a theory), 456  
adequate for **Reich**(*Th*), 607  
adjoint pair of functors, 1091  
adjoint situation, 1091  
affine geometry, 1040  
affine structures, 988  
affine transformation, 152  
affinity of space-time, 405  
Alexandrov-Zeeman theorem, 1156  
Alexandrov-Zeeman Theorem, cf. e.g. Goldblatt [108, Appendix B], 170  
algebra, 785  
algebraic element of  $\mathfrak{F}$ , 834  
Algebraic Logic, 1105  
algebraic structure, 160  
analytic geometry = represented geometry i.e. Cartesian geometry, 1106  
Archimedean field = Archimedean ordered field, 66  
Archimedean geodesic, 1182  
Archimedean ordered field, 446  
Archimedean ordered field, see footnote 88 on p, 66  
Archimedean ordered group, 1185  
art-sim hull of  $\mathbf{K}$ , 570  
art-sim models of a theory, 570  
art-sim version of a model, 569  
artificial-simultaneity version of a model, 569  
automorphism, 65, 160, 298  
auxiliary axioms, 135, 297  
auxiliary relation, 964  
axiom of choice, 434  
axiom of cone-smoothness, 519  
axiom of continuity, 766  
axiom of disjoint windows  
    (**Ax(diswind)**), 812  
axiom of functionality, 768  
B-submodel of a frame model, 708  
backwards, 471  
base for a topology, see footnote 710, 809  
basic equidistance ( $eq_0$ ) of an observer-independent geometry, 793  
basic orthogonality ( $\perp_0$ ) of an observer-independent geometry, 791  
basic paradigmatic effects, 635  
between, 530  
betweenness ( $Bw$ ), a ternary relation of an observer-independent (or a relativistic) geometry, 790  
big universe of a many-sorted model, 929  
binary relation, 26  
bisecting segments, 167  
bodies, 29  
Boolean (topological) spaces, 1018  
Boolean algebras, 1017  
boundary, 510  
boundary point, 510  
bounded, 509  
  
 $C^*$ -algebras, 1100  
Carnap, 781  
Cartesian geometry over a field, 41, 1138  
category, 1085  
category of theory morphisms, 1025  
category theoretic convention for introducing functions like  $f(-)$  or  $g(-, p)$ , 1095  
causality pre-ordering relation,  $\prec$ , 789  
changing simultaneities, 566  
circle, cf. footnote 184, 176

class form of the axiom of choice, 983  
clocks get out of synchronism, 90, 96, 436  
clocks slow down, 90, 92, 436  
clopen sets of a topology, 1017  
closed set of a topology, 870  
closure operator up to isomorphism, 1013  
closure operator, cf. footnote 996 on p, 1013  
CMA-lattices = complemented modular algebraic lattices, 1099  
codomain of a morphism, 1085  
collinear points, 167, 168  
collinearity relation (ternary relation on points), 167, 168  
collineation, 65  
common generalization of axiom systems or theories, 434  
compact subset of a topology, cf. footnote 1104 on p, 1100  
compact topological space, cf. footnote 1008 on p, 1018  
complemented modular algebraic lattices = CMA-lattices, 1099  
complete ordered field, 520  
complete theory, 301  
composition  $\circ$  of a category, 1085  
conceptual analysis of relativity, 469, 522  
conceptual analysis, cf. footnote 2 on p, 8  
concrete class of structures, see footnote 643 on p, 786, 799  
cone-smooth, 519  
cone-smooth version of a theory, 521  
congruence transformation, 350  
connectedness, ( $\sim$ ) a binary relation on points of relativistic geometries, 818  
conservative extension, 704  
continuity, 766  
continuous, 536  
continuous weak geodesic, 1183  
conventional, 581  
convergence, 791  
convex, 509  
convex hull, 509  
convex hull of a set of points of a relativistic geometry, 838  
coordinate-system, 32, 470  
coordinatization (of an ordered Pappian affine geometry), 1051  
coproduct of topologies = sum of topologies, 870  
core part of a theory, 465  
core theory part of a theory, 454  
coreflection, 1091  
coreflection arrow, 1091  
curve, 1180  
deductively closed theories, 451  
definability, 943  
definability of topological spaces, 809  
definable, 945  
definable implicitly, 936  
definable implicitly up to isomorphism, 934, 935  
definable implicitly with parameters, 935  
definable implicitly without taking reducts, 934, 935  
definable in, 930, 951  
definable over, 930, 951  
definable relation, 945  
definitional expansion, 946, 950, 951  
definitional expansion without taking reducts, 950

definitionally equivalent, 970  
 definitionally equivalent languages, cf.  
     footnote 959 on p, 991  
 definitionally equivalent theories, 972  
 definitionally equivalent, weakly, 986  
 derivative  $f'$  of  $f$ , 517  
 Desargues Theorem, cf. e.g.  
     Hilbert [134], 170  
 direction, 470  
 directional speed, 534  
 disjoint unions of frame models, 868,  
     869  
 disjoint unions of geometries, 871, 874  
 disjoint unions of non-body-disjoint  
     models, 869  
 disjoint unions of non-disjoint geome-  
     tries, 874  
 distance, 609  
 distance between simultaneities, 444  
 distance preserving transformation,  
     350  
 divisible geodesic, 1182  
 division ring, 1040  
 domain of a morphism, 1085  
 drawing compositions of world-view  
     transformations, 744  
 duality theory, duality theories, 1003–  
     1007, 1014–1027, 1069, 1096–  
     1107  
  
 Einstein's 1/2-simultaneity, 582  
 Einstein's Special Principle of Relativ-  
     ity, 123, 454  
 elementarily-equivalent, 303  
 elementary classes, 451  
 ellipse, 176  
 embeddable, 1008  
 emit a photon, 472  
 empty model, cf. footnote 977 on p,  
     1006  
 empty point, 510  
 equidistance ( $eq$ ), a 4-ary relation of  
     an observer-independent (or a  
     relativistic) geometry, 793  
 equivalence of categories, 1094  
 equivalent categories, 1094  
 Euclid's axiom, 1040  
 Euclidean case, 1135  
 Euclidean distance, 609  
 Euclidean field = Euclidean ordered  
     field, 91  
 Euclidean geometry over  $\mathfrak{F}$ , 1129  
 Euclidean length, 100, 189  
 Euclidean ordered field, 91  
 Euclidean orthogonality, 172  
 Euclidean reduct of a relativistic ge-  
     ometry, 1148  
 Euclidean straight line, 45  
 Euclidean topology, 841  
 evaluation, 945, 963  
 evaluation, cf. footnote 1134 on p, 1110  
 event, 32, 787  
 existential version of the twin paradox,  
     460  
 expansion, 153  
 expansion of a class of models, 933  
 expansion of a model, 930  
 explicit definability with parameters,  
     950  
 explicit definition, 946, 950  
 explicit definition of  $\mathfrak{N}$  over  $\mathfrak{M}$ , 950  
 explicit definition of type (1), 946  
 explicit definition of type (2.1), 947  
 explicit definition of type (2.2), 950  
 explicit definitional expansion, 946,  
     951  
 explicitly definable, 945, 950  
 explicitly definable in, 951

explicitly definable over, 951  
 explicitly definable relation, 945  
 explicitly definable without taking  
     reducts, 950  
 explicitly rigidly definable, 951  
  
 field, 30  
 field reduct, 30  
 field-extension, 1014  
 finite, 447  
 finite elements of a non-Archimedean  
     field, 447  
 finitely nr-implicitly definable, 944  
 first-order definable meta-function,  
     983  
 first-order formulation of  $\mathbf{Bax}_+^{--}$ , 542  
 first-order language, cf. language, 27  
 first-order logic, 27  
 FOL, 7  
 forwards, 470  
 Fourier transformation, 1103  
 frame model, 35  
 frame theory, 35  
 frame-language, 29, 34  
 Friedman, 774, 776, 777  
 Friedman's conceptual analysis of rel-  
     ativity, 469  
 FTL, 110  
 full Reichenbachian version, 576  
 function, 26  
 function notation  $f(-)$ ,  $\text{expr}(-) \stackrel{\text{def}}{=} \langle \text{expr}(x) : x \in A \rangle$ , 1095  
 functionality, 768  
 functor, 1085  
  
 Gödel's rotating universe, 775, 781,  
     912, 1120  
 Galilean transformation, 439  
 Galois connection, 453, 1080  
 Galois group, 1014, 1027  
  
 Galois theory of Cylindric algebras,  
     1027  
 Galois theory of fields, 1014  
 gapy in  $\mathfrak{F}$ , 834  
 general case, 1136  
 general models, 269  
 generalization of an axiom system or a  
     theory, cf. footnote ?? p, 423  
 generalized definitional expansion,  
     950, 951  
 generalized disjoint unions of frame  
     models, 869  
 generalized Galilean transformation,  
     439  
 generalized manifold, 788  
 generalized Minkowski distance, 728  
 generalized Minkowski model, 726  
 generalized Poincaré transformation,  
     728  
 generalized rotation model, 730  
 globftl, 686  
 $(\mathcal{G}, \mathcal{M})$ -duality, 1009, 1012  
 Goldblatt-Tarski reduct  $GT_{\mathfrak{M}}$  of  $\mathfrak{G}_{\mathfrak{M}}$ ,  
     923  
 $(\mathcal{G}o, \mathcal{M}o)$ -duality, 1072  
  
 half-line in a geometry  $\langle Mn; Bw \rangle$  (de-  
     fined by  $Bw$ ), 1045  
 half-line in a relativistic geometry (de-  
     fined by  $L$  and  $Bw$ ), 891  
 half-line with origin  $o$  and containing  
      $e$  (defined by  $Bw$ ), 1045  
 half-line with origin  $o$  and containing  
      $e$  (defined by  $L$  and  $Bw$ ), 891  
 Hausdorff (i.e.  $T_2$  space), cf. foot-  
     note 1009 on p, 1018  
 higher-order logic, 801  
 homeomorphism, see footnote 676 on  
     p, 798

homogeneity, 405  
 homogeneity of space, 405  
 homogeneity of space-time, 405  
 homomorphism, 160, 298  
 hyper-plane, 164, 566  
 hyper-plane in a relativistic geometry, 1129  
  
 ideal of infinitely small numbers in an ordered field, 447  
 imaginary observer, 803  
 implicit definition, 934, 936  
 implicitly definable, 934, 936  
 implicitly definable without taking reducts, 935  
 incidence geometry associated to  $\mathfrak{M}$ , 1174  
 incidence relation, 789  
 independent axiom system, 75  
 independent subset of  $Mn$ , 1039  
 inertial bodies, 31  
 infimum, 451  
 infinitely large, 447  
 infinitely small, 447  
 instances of SPR, 349  
 interior, 510  
 interpret, 968  
 interpretation of one theory (or language) in another, Fig.306, 984, 1020, 1021, 1023  
 interval of  $\mathbf{F}_0$ , 1179  
 isometry, 134, 349  
 isomorphism, 298  
 isomorphism as a distinguished morphism of a category, 1093  
 isomorphism between observer-independent (or relativistic) geometries, 798  
 isotropy, 219, 399, 468  
  
 Kant, 775, 781  
  
 language = first-order language = a language of first-order logic = similarity type, cf. any textbook on logic e.g. Monk [197, p.14] or Enderton [82], 27  
 language of first-order logic, 27  
 language, language of  $\mathbf{K}$ , cf. also footnote 924, 962  
 Laplace transform, 1101  
 Laplace transformation, 1101  
 lattice of theories, 451  
 lattice, cf. footnote 1002 (cf. also p.451), 1015  
 laws of nature, 348, 777, 778, 1107  
 least common generalization of theories, 452  
 Leibniz, 775  
 Leibniz's principle of identity of indistinguishable concepts, cf. footnote 621 on p, 775  
 life-line, 47  
 light-cone, 207, 473  
 light-cone,  $\text{Cone}_{m,p}$ , 473  
 light-point, 510  
 light-sphere, 505  
 limit, 791  
 limit of lines, 791  
 limit of sequences, 791  
 linear operator, cf. footnote 1111 on p, 1102  
 linear transformation, 65  
 linearly ordered field, 30, 43  
 lines, 29, 30  
 $\text{Lines} = L$   
 cf. footnote 958 on p, 991  
 lines ( $L$ ) of an observer-independent (or a relativistic) geometry,

789

lines, see  $L$ , *lines*, Eucl e.g. on p, 789

local definability, 937, 943

locally additive geodesic, 1183

locally compact topological space, 1100

location, 32

locfl, 686

logic, cf. item 1.1.(XI) and footnote 37 on pp, 13, 29

logical positivism, 774, 781

Lorentz Group, 408

Lorentz transformation, 152

Lorentzian metric, see footnote 668 on p, 797

manifold, 788

many-sorted approximation of higher-order logic see higher-order logic, 801

many-sorted first-order language, 27

many-sorted first-order logic, 27

many-sorted logic, 27

many-sorted logic, cf. item 1.1.(XI) and footnote 37 on pp, 13, 29

many-sorted model theory, 298

many-sorted models, cf. item 1.1.(XI) and footnote 37 on pp, 13, 29

many-sorted universal algebra, 298

mathematical logic, cf. item 1.1.(XI) and footnote 37 on pp, 13, 29

maximal definitional expansion, 951

maximal geodesic, 1182

median observer, 306

meter rods shrink, 436

$(\mathcal{M}, \mathcal{G})$ -duality, 1009, 1012

midpoint of a segment, 166

Minkowski distance, 728, 860

Minkowski model, 331, 726

Minkowski-circles, 89

Minkowski-distance, 152

Minkowski-orthogonal lines, 860

Minkowski-sphere, 89

Minkowskian case, 1135

Minkowskian geometry, 859

Minkowskian geometry,  $\perp_0$ -version, 878

Minkowskian orthogonality ( $\perp_\mu$ ), 860

mirror image w.r.t. a line, 173

model theory, 160, 298

$(\mathcal{M}o, \mathcal{G}o)$ -duality, 1072

morphisms of a category, 1085

move backwards in direction  $d$ , 471

move forwards in direction  $d$ , 470

move in direction  $d$ , 470

natural number, 26

neighborhood, 189, 536

Newtonian kinematics, 423

nice automorphisms of  $\mathfrak{G}_m$ , 915

non-adequate, 607

non-body-disjoint models, 870

non-elementarily-equivalent models cf. elementarily-equivalent, 303

non-standard higher-order logic, 801

non-standard higher-order logic = many-sorted approximation of higher-order logic, see higher-order logic, 801

non-uniform definability, 943

nonstandard simultaneities, 554

nonstandard synchronization, 554

not adequate (a symmetry principle being not adequate for a theory), 455

nr-implicitly definable, 936

objects of a category, 1085

observational, 774

observational/theoretical, 774  
 observer brothers, 702  
 observer-dependent geometry, 880  
 observer-independent geometry  $\mathfrak{G}_m$ ,  
     786, 787  
 observer-point, 510  
 observers, 31  
 Occam, 775  
 Occam's razor, 123, 464  
 Occam's razor, cf. footnote 621 on p,  
     775  
 of nonzero speed, world-view transfor-  
     mation, 709  
 one-by-one (explicit) definability, 951  
 one-by-one definability, 937, 943  
 one-by-one nr-implicitly definable, 943  
 one-sorted vector space, 42  
 one-way speed of light, 553  
 open interval,  $\text{int}(p, q)$ , 509  
 open set of a topology, 870  
 ordered field, 30, 43  
 ordered field corresponding to an or-  
     dered Pappian affine geometry,  
     1049  
 ordered field reduct of a frame model,  
     30, 43  
 ordered Pappian affine geometry, 1044  
 orthogonal, 172  
 orthogonal in the Euclidean sense, 172  
 orthogonality ( $\perp_r, \perp$ ), a relation of an  
     observer-independent (or a rel-  
     ativistic) geometry, 790  
 orthogonality, Minkowskian ( $\perp_\mu$ ), 860  
 orthogonality, relativistic, 790  
  
 P1, 522  
 P2, 522  
 P3, 522  
 Pappian affine geometry, 1042  
  
 Pappus-Pascal Property, 1041  
 paradigmatic effects, 90, 491, 635  
 paradigmatic theorems, 90  
 parallel lines, 1039  
 parallel lines ( $\ell \parallel_{\mathfrak{G}} \ell_1$ ) in an observer-  
     independent (or in a relativis-  
     tic) geometry, 790  
 parallel straight lines, 164  
 parallelogram, 167  
 parameterization, 1180  
 parametrizable curve, 1180  
 parsimonious version of a theory, 465  
 partial derivative,  $\partial_i f$ , 518  
 partial function, 517  
 parts of SPR, 349  
 Pasch's Law, 1043  
 periodical body, 1117  
 photon-disjoint unions of frame mod-  
     els, 869  
 photon-free reduct of a frame model,  
     708  
 photon-free sub-reduct of a frame  
     model, 708  
 photon-glued disjoint unions of geome-  
     tries, 873, 874  
 photon-glued disjoint unions of non-  
     disjoint geometries, 874  
 photon-like Archimedean geodesic,  
     1179  
 photon-like geodesic, 1179  
 photon-like lines,  $L^{Ph}$ , 788  
 photon-like quasi geodesic, 1179  
 photon-like separated events,  $e \equiv^{Ph} e_2$ ,  
     799  
 photon-line, 58  
 photon-preserving affine transforma-  
     tions, 265  
 photon-preserving transformation, 62  
 photon-sphere,  $\mathbb{C}_m$ , 505

photons, 31  
 plane, 164  
 plane generated by a set of points of a relativistic geometry, 819  
 plane tangent to a light-cone in a relativistic geometry, cf. footnote 799 on p, 857  
 Poincaré Group, 408  
 Poincaré transformation, 153, 728  
*Points = Mn*  
     cf. footnote 958 on p, 991  
 points (*Mn*) of an observer-independent geometry, 787  
 points of a topology, 870  
 poset, 451, 1097  
 potential laws (of nature), 777, 778, 1107  
 pre-standard configuration, 602  
 pre-standard symmetric configuration, 603  
 pre-standard-sym configuration, 603  
 prime ideal, cf. footnote 1002 on p, 1015  
 principle of absolute space, 438  
 principle of absolute time, 437  
 principle of isotropy, 219  
 principle of parsimony, 123, 464, 775  
 principles (\*1), (\*2), (\*3), 710  
 projection functions, 947  
 pseudo-metric reduct of a relativistic geometry, 1179  
 pseudo-metric, *g*, 797  
  
 quantities, 29  
 quasi geodesic, 1181  
 quasi-standard configuration, 605  
 quasi-standard-sym configuration, 605  
  
 rank of a relation symbol, 931  
 real-closed field, 301  
  
 reduct, 30  
 reduct of a class of models, 932  
 reduct of a model, 930, 932  
 reference frames, 54  
 reflection, 1090, 1091  
 reflection arrow, 1091  
 reflection w.r.t. a line, 173  
 Reichenbach, 774, 775, 777, 781  
 Reichenbach-adequate, 608  
 Reichenbachian version of a theory, 562  
 Reichenbachian version of the observer-independent geometry, 799  
 Reichenbachizing a theory, 563  
 relation of connectedness,  $\sim$ , 818, 894  
 relative space-time, 470  
 relativistic addition of velocities, 420  
 relativistic effect, 491  
 relativistic geometry, 798  
 relativistic orthogonalities,  $\perp'_r$ ,  $\perp''_r$  (alternatives for  $\perp_r$ ), 810  
 relativistic orthogonalities,  $\perp'''_r$ ,  $\perp^\omega_r$  (alternatives for  $\perp_r$ ), 821  
 relativistic orthogonality,  $\perp_r$ ,  $\perp$ , 790, 792  
 relativity based on Einstein's SPR only (i.e. not mentioning speed of light), 704  
 relativity without any connection with electrodynamics, 704  
 relativized model, 569  
 relativized models of a theory, 570  
 relativizing with artificial simultaneities, 569  
 representation theorems, 1106  
 residuated-residual pair, 1097  
 rhombus, 172  
 rhombus transformation, 72

rigidly definable, 951  
 Robb hyper-plane, 804, 1130  
 Robb plane, 1163  
 Robb's "after", 1156  
 rotating observers, 554  
 rotation model, 729  
 rotation, cf. footnote 584 on p, 729  
 round-trip of photons, 553  
  
 segment, 166  
 semantical, 581  
 sentence, see footnote 954 on p, 987  
 short geodesic, 1182  
 short space-like geodesic, 1182  
 short time-like geodesic, 1180  
 similar algebraic structures, cf. footnote 167 p, 161  
 similar models, 932  
 similar models, cf. footnote 167 p, 161  
 similar structures, 798  
 similarity type, 27  
 simple models, 249  
 simplifying principles part of a theory, 465  
 simultaneity-stable, 578  
 simultaneous events, 95  
 slow observer, 483  
 slow-line, 58  
 space component, 32, 43  
 space coordinates, 43  
 space location, 32  
 space part, 54  
 space part of the coordinate-system, 208  
 space-axes, 470  
 space-like Archimedean geodesic, 1181  
 space-like geodesic, 1181  
 space-like hyper-plane in a relativistic geometry, 1130  
  
 space-like lines,  $L^S$ , 788  
 space-like quasi geodesic, 1180  
 space-like separated events, 799  
 space-part,  $S$ , 470  
 space-time, 33, 470, 788, 802  
 space-time location, 32  
 spaceships shrink, 90, 100  
 spatial direction, 470  
 special case of an axiom system or a theory, cf. footnote ?? p, 423  
 special relativity with no photons, 704  
 speed, 47  
 speed of a body in a direction, 534  
 speed of light axiom, 564  
 speed of light axioms, 555  
 speed of light is finite, 491  
 SPR, 123, 347, 454, 523  
 SPR<sub>0</sub>, 523  
 square of generalized Minkowski distance, 728  
 standard configuration, 71  
 Standard Lorentz Group, 408  
 standard Lorentz transformation, 152  
 standard model of **Reich**<sub>0</sub>(*Th*), 570  
 standard models of a Reichenbachian theory, 570  
 Stone, 786, 1015, 1019  
 Stone duality theory, 1015, 1019  
 Stone representation theorem, cf. also Stone duality theory, 786  
 straight line, 45  
 streamlined partial metric = time-like-metric, 1169  
 streamlined partial metric reduct of  $\mathfrak{G}_m$ , 1170  
 strict standard configuration, 71, 709  
 strict standard world-view transformation, 709  
 strong embedding, 1085

strong geodesic, 1182  
 strong submodel, 161  
     cf. footnote 983 on p, 1008  
 strong substructure, cf. strong sub-  
     model, 161  
 strongly continuous, 537  
 strongly non-inertial body, 1119  
 sub-vector space, cf. footnote 166 on p,  
     160  
 sub-vocabulary, 932  
 subalgebra, 160  
 subbase for a topology, cf. foot-  
     note 1004 on p, 1016  
 subcategory, 1090  
 submodel, 161  
 subspace, cf. footnote 166 on p, 160  
 substructure, 161  
 sum of topologies = coproduct of  
     topologies, 870  
 sum topology, cf. footnote 766 p, 841  
 supremum, 451  
 surface, 594  
 symmetric version of **Reich(Basax)**,  
     627  
 symmetric version of a theory, 454, 455  
 symmetry, 347  
 symmetry axioms, 123, 844, 910, 913,  
     1107  
 symmetry principle, 454, 455, 777  
 symmetry principle part of a theory,  
     454  
 synchronism, 90  
 syntactical, 581  
 syntax-semantics duality, 453, 1020  
 (syntax, semantics)-duality, 1020  
 synthetic geometry = “axiomatic ge-  
     ometry”, 1106  
 $T_2$  (i.e. Hausdorff) space, cf. foot-  
     note 1009 on p, 1018  
 $T_0$ -space,  $T_0$  topological space, cf.  
     footnote 1006 on p, 1017  
 taking limits, 791  
 terminology of universal algebra, 160,  
     161  
 theorem-schemas (A)–(I) for duality  
     theories, 1012  
 theoretical, 774  
 theory, 299  
 theory generated by an axiom system,  
     1027, 1111  
 theory morphisms, 1025  
 three-sorted first-order language = 3-  
     sorted first-order language, 27  
 three-way speed of light, 572  
 time axis, 43  
 time component, 32, 43  
 time coordinate, 43  
 time-axis, 470  
 time-like Archimedean geodesic, 1181  
 time-like geodesic, 1181  
 time-like hyper-plane in a relativistic  
     geometry, 1130  
 time-like lines,  $L^T$ , 788  
 time-like quasi geodesic, 1180  
 time-like separated events,  $e \equiv^T e_1$ ,  
     799  
 time-like-metric  $g^\prec$ , 1169  
 time-like-metric geometry, 1170  
 time-like-metric reduct of a relativistic  
     geometry, 1170  
 time-like-metric relativistic geometry,  
     1170  
 time-like-metric structure, 1170  
 time-part,  $\bar{t}$ , 470  
 topology generated by ..., see foot-  
     note 672 on p, 797  
 topological space, 870

topology  $(\mathcal{T})$  of  
     an observer-independent (or a  
     relativistic) geometry, 797  
 trace, 47  
 transformation, 62  
 translation, 152  
 twin paradox, 38, 140, 460  
 two-dimensional plane in a relativistic  
     geometry, see footnote 798 on  
     p, 857  
 two-sorted vector space, 42  
 two-way speed of light, 553  
 type A transformations, 751  
 type B transformations, 751  
 type C transformations, 751  
 types A, B, C, 751  
  
 ultrapower, 1139  
 uniform (explicit) definability, 951  
 uniform implicit definition, 937  
 uniformly definable, 937  
 unions of geometries and models, 868  
 unit vector, 69  
 universal algebra, 160, 161, 298  
 universes of a three-sorted model, 33  
  
 vector, 38  
 vector space, 42  
 velocities add up, 437  
 velocity, 48  
 velocity structure, 420  
 velocity-equivalence, 421  
 visibility relation, 192  
 vocabulary, 27, 931, 962  
 vocabulary of a model, 931  
  
 weak geodesic, 1183  
 Weak Principle of Isotropy, 219  
 weak submodel, 1008  
 weakly definitionally equivalent, 986  
  
 weakly standard configuration, 416  
 why-type questions, 469  
 window, 876  
 witness to  $eq_0^S$ , 902  
 witness to  $eq_0$ , 899  
 work (configurations work), 604  
 world-view, 52  
 world-view function, 32  
 world-view relation, 32  
 world-view transformation, 56, 913–  
     915  
 world-view transformation of nonzero  
     speed, 709  
 world-view transformations of types A,  
     B, C, 751  
 WPI, 219

## References

- [1] J. Adámek. *Theory of mathematical structures*. Reidel, Dordrecht, Boston, Lancaster, 1983.
- [2] J. Adámek, H. Herrlich, and G.E. Strecker. *Abstract and Concrete Categories*. A Wiley-interscience publication. John Wiley & Sons, Inc, New York, 1990.
- [3] J. Adámek and S. Mac Lane, editors. *Categorical Topology and its relation to analysis, algebra and combinatorics*. World Scientific, Singapore, New Jersey, London, Hong Kong, 1989.
- [4] A.D. Alexandrov. On Lorentz transformations. In *Sessions of the Mathematical Seminar of the Leningrad Section of the Mathematical Institute*, 15 September 1949. (abstract, in Russian).
- [5] A.D. Alexandrov. A contribution to chronogeometry. *Canadian J. Math.*, 19:1119–1128, 1967.
- [6] A. Andai. Category theoretic concepts for physics. available from [www.math.bme.hu/~andai/kutatas/kut.html](http://www.math.bme.hu/~andai/kutatas/kut.html).
- [7] A. Andai. Lorentz-transformation for tachyons. Manuscript, Eötvös Loránd University Budapest, 2000.
- [8] A. Andai. On the mathematical foundations of quantum mechanics (A kvantummechanika matematikai alapjairól). Master's thesis, Eötvös Loránd Univ., Budapest, 1998. (in Hungarian) available from [www.math.bme.hu/~andai/kutatas/kut.html](http://www.math.bme.hu/~andai/kutatas/kut.html).
- [9] H. Andréka, S.D. Comer, and I. Németi. Epimorphisms in cylindric algebras. Manuscript, 1983.
- [10] H. Andréka, S.D. Comer, and I. Németi. Surjectivness of epimorphisms of cylindric algebras. *Absts. Amer. Math. Soc.*, 4, 1983.
- [11] H. Andréka, T. Gergely, and I. Németi. Model theoretical semantics for many-purpose languages and language hierarchies. *Computational linguistics* (Proc. 8th Intern. Conf. Tokio), pages 213–219, 1980.
- [12] H. Andréka, T. Gergely, I. Németi, and I. Sain. Theory morphisms, step-wise refinement of program specifications, representation of knowledge, and cylindric algebras. Preprint, Mathematical Institute Budapest, 1980.

- [13] H. Andréka, S. Givant, and I. Németi. Decision problems for varieties of relation algebras. *Memoirs of the AMS*, 126(604):126+xiv pages, March 1997.
- [14] H. Andréka, V. Goranko, Sz. Mikulás, I. Németi, and I. Sain. Effective first order temporal logics. In *Time and Logic, a computational approach*, pages 51–129. Ed. by L. Bolc and A. Szalas. UCL Press Ltd., London, 1995.
- [15] H. Andréka, R.J. Greechie, and Strecker G.E. On residuated approximations. In *Categorical Methods in Computer Science (With aspects from Topology)*, volume 393 of *Lecture Notes in Computer Science*, pages 333–339. Springer-Verlag, Berlin, 1989.
- [16] H. Andréka, J.X. Madarász, and I. Németi. Logical analysis of special relativity. In J. Gerbrandy, M. Marx, M. de Rijke, and Y. Venema, editors, *JFAK. Essays Dedicated to Johan van Benthem on the Occasion of his 50th Birthday*, <http://www.wins.uva.nl/~j50/cdrom>, 1999. Vossiuspers, Amsterdam University Press. CD-ROM, ISBN: 90 5629 104 1.
- [17] H. Andréka, J.X. Madarász, and I. Németi. Decidability, undecidability and Gödel incompleteness in relativity theories. Technical report, A. Rényi Institute of Mathematics, Budapest, 2001.
- [18] H. Andréka, J.X. Madarász, and I. Németi. On the logical structure of relativity theories. Research report, Alfréd Rényi Institute of Mathematics, Hungar. Acad. Sci., Budapest, 2001. With contributions from A. Andai, G. Sági, I. Sain and Cs. Tőke. <http://www.math-inst.hu/pub/algebraic-logic/Contents.html>.
- [19] H. Andréka, J.X. Madarász, and I. Németi. with contributions from A. Andai, G. Sági, I. Sain and Cs. Tőke. On the logical structure of relativity theories, 2001. Future edition of [18].
- [20] H. Andréka, J.X. Madarász, and I. Németi. Spaceships get distorted in Reichenbachian theories, December 1999.
- [21] H. Andréka, J.X. Madarász, and I. Németi. Defining new universes in many-sorted logic, January 2001. A. Rényi Institute of Mathematics, Budapest.
- [22] H. Andréka, J.X. Madarász, and I. Németi. A note on definability and definitional equivalence in model theory, June 2000. A. Rényi Institute of Mathematics, Budapest.

- [23] H. Andréka, J.X. Madarász, I. Németi, and G. Sági. Accelerated observers, a continuation of On the logical structure of relativity theories, in preparation. A. Rényi Institute of Mathematics, Budapest.
- [24] H. Andréka, J.X. Madarász, I. Németi, G. Sági, and I. Sain. Analyzing the logical structure of relativity theory via model theoretic logic. Lecture notes for A'dam course., 1998 December 12. <http://www.math-inst.hu/pub/algebraic-logic/relativity1.ps>, [relativity2.ps](http://www.math-inst.hu/pub/algebraic-logic/relativity2.ps).
- [25] H. Andréka, J.X. Madarász, I. Németi, G. Sági, and I. Sain. Analyzing the logical structure of relativity theory via model theoretic logic. Lecture notes for A'dam course., 1998 early March. (This was distributed at CCSOM of Univ. of Amsterdam).
- [26] H. Andréka and Sz. Mikulás. Lambek calculus and its relational semantics: completeness and incompleteness. *Journal of Logic, Language and Information*, 3(1):1–38, 1994.
- [27] H. Andréka and I. Németi. Generalization of variety and quasivariety-concept to partial algebras through category theory. *Dissertationes Mathematicae (Rozprawy Math)*, 204:1–56, 1976.
- [28] H. Andréka, I. Németi, and I. Sain. Craig property of a logic and decomposability of theories. In *Proceedings of the 9th Amsterdam Colloquium*. Institute for Logic Language and Computation, 1993.
- [29] H. Andréka, I. Németi, and I. Sain. Applying algebraic logic to logic. In T. Rus M. Nivat, C. Rattray and G. Scollo, editors, *Algebraic Methodology and Software Technology (AMAST'93, Proceedings of the Third International Conference on Algebraic Methodology and Software Technology, The Netherlands, 21-25 June 1993)*, Workshops in Computing series, pages 5–26. University of Twente, Springer-Verlag, London, 1994.
- [30] H. Andréka, I. Németi, and I. Sain. Algebraic Logic. In *Handbook of Philosophical Logic Vol. 2, second edition*, pages 133–247. D.M. Gabbay and F. Guenther (eds), Kluwer Academic Publishers, Dordrecht, Boston, London, 2001.
- [31] H. Andréka, J.A.F.K. van Benthem, and I. Németi. Modal languages and bounded fragments of predicate logic. *Journal of Philosophical Logic*, 27:217–274, 1998.

- [32] M.A. Arbib and E.G. Manes. Adjoint machines, state-behavior machines and duality. *J. Pure Appl. Algebra*, 6:313–345, 1975.
- [33] M.A. Arbib and E.G. Manes. A categorist’s view of automata and systems. In *Lecture Notes in Computer Science*, volume 25, pages 51–64. Springer–Verlag, Berlin, 1975.
- [34] R. Audi, editor. *The Cambridge Dictionary of Philosophy*. Cambridge University Press, 1995. ISBN 0-521-48328-X (pbk.).
- [35] J. Ax. The elementary foundations of spacetime. *Found. Phys.*, 8(7–8):507–546, 1978.
- [36] J. Baez and J. Dolan. Higher-dimensional algebra and topological quantum field theory. *Jour. Math. Phys.*, 36:6073–6105, 1995.
- [37] J.C. Baez. Higher-dimensional algebra II: 2-Hilbert spaces. Research report, Department of Mathematics, University of California at Riverside, California, USA, 1996.
- [38] P. Bahl, J. Cole, N. Galatos, P. Jipsen, and C. Tsinakis. Cancellative residuated lattices. Research report, Department of Mathematics, Vanderbilt University, Nashville, 2001.
- [39] J.B. Barbour. *Absolute or relative motion?* Cambridge University Press, Cambridge, 1989.
- [40] M. Barr and C. Wells. *Toposes, Triples and Theories*. A Series of Comprehensive Studies in Mathematics. Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1985.
- [41] J. Barwise. *Admissible sets and structures*. Springer–Verlag, Berlin, 1975.
- [42] J. Barwise. *Handbook of Mathematical Logic*. North-Holland, Amsterdam, 1977.
- [43] J. Barwise and S. Feferman, editors. *Model-Theoretic Logics*. Springer–Verlag, Berlin, 1985.
- [44] J. Bell and M. Machover. *A course in mathematical logic*. North–Holland, Amsterdam, 1977.
- [45] J.L. Bell and A.B. Slomson. *Models and ultraproducts*. North-Holland, Amsterdam, 1969.

- [46] N. Belnap. Branching space-time. *Synthese*, 92:385–434, 1992.
- [47] G. Birkhoff. *Lattice theory*, volume 25 of *Colloq. Publ.* Amer. Math. Soc., 1967.
- [48] P. Blackburn, M. de Rijke, and Y. Venema. *Modal Logic*. Kluwer, to appear.
- [49] J. Blészer, P. Gnädig, and Zs. Varga. Benne van-e az elektrodinamika törvényeiben a relativitáselmélet? Dept. Physics, ELTE University, Budapest, 2001.
- [50] F. Borceaux. *Handbook of Categorical Algebra 1. Basic Caterory Theory*. Cambridge University Press, 1994.
- [51] O. Bratteli and D.W. Robinson. *Operator algebras and quantum statistical mechanics 1*. Springer-Verlag, Berlin, 1979, 1987.
- [52] W.L. Burke. *Spacetime, geometry, cosmology*. Books in Astronomy. University Science Books, Mill Valley, California, 1980.
- [53] P. Burmeister. *A model theoretic oriented approach to partial algebras*. Akademi-Verlag, Berlin, 1986.
- [54] S. Burris and H.P. Sankappanavar. *A course in universal algebra*. Graduate Texts in Mathematics. Springer Verlag, New York, 1981.
- [55] H. Busemann. *The geometry of geodesics*. Academic Press, New York, 1955.
- [56] H. Busemann. *Time like spaces*. Dissertationes Mathematicae (Rozprawy Math). Mathematical Institute of Polish Academy of Sci., Warsaw, 1967.
- [57] R. Carnap. *The logical structure of the world*. University of California Press, Berkeley, 1967. Translated by R. G. George (Original German edition published in 1928).
- [58] G.J. Chaitin. *The Limits of Mathematics – Course Outline & Software*. IBM, Watson Center, Yorktown Heights, December 12, 1993.
- [59] C.C. Chang and H.J. Keisler. *Model Theory*. North-Holland, Amsterdam, 1973, 1990.
- [60] P.M. Cohn. *Universal algebra*. Harper and Row, New York, 1965.

- [61] S.D. Comer. Galois-theory of cylindric algebras and its applications. *Trans. Amer. Math. Soc.*, 286:771–785, 1984.
- [62] H.S.M. Coxeter. *Introduction to Geometry*. Wiley, New York, 1969.
- [63] L. Crane. Clock and category: is quantum gravity algebraic? *Jour. Math. Phys.*, 36:6180–6193, 1995.
- [64] L. Csirmaz. *Nemstenderd Analizis*. Typotex, Budapest, 1999.
- [65] G. Czédli. *Lattice Theory*. JATEPress, Szeged, 1999. (in Hungarian).
- [66] N.C.A. da Costa, F.A. Doria, and J.A. de Berros. A Suppes predicate for general relativity and set theoretically generic spacetimes. *Internat. J. Theoret. Phys.*, 29:935–961, 1990.
- [67] G. Darvas. Ontological levels and symmetry breaking. *Paideia, Philosophy of Science*, 1988. <http://www.bu.edu/wcp/Papers/Scie/ScieDarv.htm>.
- [68] B.A. Davey and H.A. Priestley. *Introduction to Lattices and Order*. Cambridge mathematical textbooks. Cambridge University Press, 1990.
- [69] Gy. Dávid. Personal communication with Németi on 1998 December 9, Math. Inst. seminar.
- [70] Gy. Dávid. Personal communication with Németi on 1998 Nov. 2.
- [71] Gy. Dávid. Relativity without speed of light. Manuscript (Dept. of General Physics, Eötvös Loránd Univ., Budapest), in preparation.
- [72] P. Davies and J. Gribbin. *The matter myth*. Penguin books, 1991.
- [73] J.W. Dawson, Jr. *Logical Dilemmas. The life and work of Kurt Gödel*. A. K. Peters, Ltd., 1997.
- [74] G.D. Dimov and W. Tholen. A characterization of representable dualities. In Adámek and Mac Lane [3], pages 336–357.
- [75] R. d’Inverno. *Introducing Einstein’s Relativity*. Oxford University Press, 1983.
- [76] F. J. Dyson. *Infinite in all directions*. Penguin Books, London, 1990.
- [77] H.D. Ebbinghaus, J. Flum, and W. Thomas. *Mathematical Logic*. Springer-Verlag, Berlin, 1984.

- [78] J. Ehlers, F.A.E. Pirani, and A. Schild. The geometry of free fall and light propagation. In *General relativity, Papers in Honor of J.L. Synge*, pages 63–84. Clarendon press, Oxford, 1972.
- [79] H. Ehrig and B. Mahr. *Fundamentals of algebraic specification 1*. Springer-Verlag, Berlin, 1985.
- [80] A. Einstein. *On the special and the general theories of relativity*. Verlag von F. Vieweg & Son, Braunschweig, 1921. in German.
- [81] A. Einstein. The foundation of the general theory of relativity. In W. Perrett and G. B. Jeffery (trans.), editors, *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity*. Dover, New York, 1923. (Original German version published in *Annalen der Physik* 49 [1916]).
- [82] H.B. Enderton. *Introduction to Mathematical Logic*. North-Holland, Amsterdam, 1972.
- [83] R. Engelking. *General topology*. PWN-Polish Scientific Publishers, Warszawa, 1977.
- [84] M. Farkas, editor. *Concise Lexicon of Mathematics*. Műszaki kiadó, Budapest, 1972. (in Hungarian).
- [85] H.H. Field. *Science without numbers*. Basil Blackwell, Oxford, 1980.
- [86] J.D. Foulis. Baer  $*$ -semigroups. *Proc. Amer. Math. Soc.*, 11:648–654, 1960.
- [87] D. Freed. Higher algebraic structures and quantization. *Commun. Math. Phys.*, 159:343–398, 1994.
- [88] R. Freese. Free modular lattices. *Transactions of the American Mathematical Society*, 261:81–91, 1980.
- [89] P.J. Freyd and A. Scedrov. *Categories, Allegories*. North-Holland, Amsterdam, New York, Oxford, Tokyo, 1990.
- [90] M. Friedman. *Foundations of Space-Time Theories. Relativistic Physics and Philosophy of Science*. Princeton University Press, 1983.
- [91] J. Fröhlich and T. Kerler. *Quantum groups, quantum categories and quantum field theory*, volume 1542 of *Springer Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 1993.

- [92] L. Fuchs. *Partially Ordered Algebraic Systems*. Pergamon Press, 1963.
- [93] D. Gabbay and F. Guentner, editors. *Handbook of Philosophical Logic*. D. Reidel Publishing Company, Dordrecht, 1984. Second edition Kluwer Academic Publishers, Dordrecht, 2001.
- [94] D.M. Gabbay, editor. *What is a Logical System?* Oxford University Press, 1994.
- [95] D.M. Gabbay. *Fibring logics*. Oxford University Press, 1998.
- [96] D. Gallin. *Intensional and Higher-Order Modal Logic*. North Holland/American Elsevier, Amsterdam-New York, 1975.
- [97] P. Gärdenfors. *Knowledge in flux*. MIT Press, Cambridge USA, 1988.
- [98] M. Gardner. *Relativity Simply Explained*. Dover, 1997.
- [99] M. Gehrke and B. Jónsson. Monotone bounded distributive lattice expansions. *Mathematica Japonica*, 52(2), 2000.
- [100] T. Gergely. Algebraic representation of language hierarchies. *Acta Cybernet.*, 5:307–323, 1980.
- [101] R. Geroch. *General relativity from A to B*. The University of Chicago Press, 1978.
- [102] S. Givant. Universal classes of simple relation algebras. *The Journal of Symbolic Logic*, 64:575–589, 1999.
- [103] Kurt Gödel. *Collected Works. Volume II. Publications 1938-1974*. Oxford University Press, 1990. Editors: S. Feferman, J.W. Dawson, Jr., S.C. Kleene, G.H. Moore, R.M. Solovay, and J. van Heijenoort.
- [104] Kurt Gödel. *Collected Works. Volume III. Unpublished Essays and Lectures*. Oxford University Press, 1995. Editors: S. Feferman, J.W. Dawson Jr., W. Goldfarb, and C. Parsons.
- [105] J. Goguen and R. Burstall. Institutions: Abstract model theory for specifications and programming. *Journal of the ACM*, 39,1:95–146, 1992.
- [106] R. Goldblatt. Diodorian modality in Minkowski spacetime. *Studia Logica*, XXXIX:219–236, 1980.

- [107] R. Goldblatt. *Topoi. The Categorical Analysis of Logic*. North-Holland, Amsterdam, New York, Oxford, 1984.
- [108] R. Goldblatt. *Orthogonality and Spacetime Geometry*. Springer–Verlag, Berlin, 1987.
- [109] R. Goldblatt. Algebraic polymodal logic: a survey. *Logic Journal of IGPL*, 8(4):393–450, 2000.
- [110] E.C. Gootman and A.J Lazar. Quantum groups and duality. *Rev. Math. Phys.*, 5:417–451, 1993.
- [111] G. Grätzer. *General lattice theory*. Akademie-Verlag, Berlin, 1978.
- [112] G. Grätzer. *Universal Algebra, Second edition*. Springer–Verlag, New York, 1979.
- [113] B. Greene. *The Elegant Universe*. W.W.Norton & Company, New York and London, 1999. ISBN 0-393-04688-5.
- [114] B. Gruber and R.S. Millman. *Symmetries in science*. Plenum Press, New York and London, 1980. ISBN 0-306-40541-5.
- [115] A. Grünbaum. Simultaneity by slow clock transport in the special theory of relativity. *Philosophy of Science*, 36:5–43, 1969.
- [116] A. Grünbaum. *Philosophical problems of space and time*. D. Reidel Publishing Co., Boston, 1974.
- [117] R. Guitart. On the geometry of computations. II. *Cahiers de Topologie et Géométrie Différentielle Catégoriques*, XXIX(4):297–326, 1988.
- [118] P. Hájek, editor. *Gödel’96 (Logical Foundations of Mathematics, Computer Science and Physics - Kurt Gödel’s Legacy, Brno, Czech Republic August 1996, Proceedings)*. Lecture Notes in Logic Vol. 6, Springer–Verlag, Berlin, 1996.
- [119] P. Hájek and P. Pudlák. *Mathematics of First Order Arithmetic*. Springer–Verlag, Berlin, 1993.
- [120] Gy. Hajós. *Bevezetés a geometriába*. Tankönyvkiadó, Budapest, 1971. In Hungarian.

- [121] Gy. Hajós and J. Strohmajer. *A geometria alapjai* (The basics of geometry). Technical report, Eötvös Loránd University, Budapest, 1972. Tankönyvkiadó, Budapest (Hungary); (in Hungarian).
- [122] P.R. Halmos. *Finite-dimensional vector spaces*. Springer-Verlag, Berlin, 1974.
- [123] G. Hansoul. A duality for Boolean algebras with operators. *Algebra Universalis*, 17:34–49, 1983.
- [124] S.O. Hansson. *A textbook of belief dynamics*. Kluwer Academic Publishers, 1999.
- [125] M. Hausner. *A vector space approach to geometry*. Prentice-Hall Inc., Englewood Cliff N. J., 1965.
- [126] S.W. Hawking and G.F.R. Ellis. *The large scale structure of space-time*. Cambridge University Press, 1973.
- [127] G. Helzer. Special relativity with acceleration. *American Mathematical Monthly*, 107:219–237, 2000.
- [128] L. Henkin. Completeness in the theory of types. *Journal of Symbolic Logic*, 15:81–91, 1950.
- [129] L. Henkin, J.D. Monk, and A. Tarski. *Cylindric Algebras Parts I and II*. North-Holland, Amsterdam, 1971 and 1985.
- [130] L. Henkin, J.D. Monk, A. Tarski, H. Andréka, and I. Németi. *Cylindric Set Algebras*, volume 883 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, Heidelberg, New York, 1981.
- [131] L. Henkin, P. Suppes, and A. Tarski, editors. *The axiomatic method with special reference to geometry and physics*, Studies in Logic and the Foundations of Mathematics. North-Holland, Amsterdam, 1959.
- [132] N.J. Hicks. *Notes on differential geometry*. Van Nostrand Co. Inc., 1965.
- [133] D. Hilbert. *Grundlagen der Geometrie*. Leipzig / B. G. Teubner Verlag, Stuttgart, 1899/1977.
- [134] D. Hilbert and P. Bernays. *Grundlagen der Geometrie*. Teubner Verlag, Stuttgart, 1956.

- [135] R. Hirsch and I. Hodkinson. *Relation algebras by games*. North-Holland, to appear.
- [136] W. Hodges. *Model Theory*. Cambridge University Press, 1993.
- [137] W.A. Hodges, I. Hodkinson, and H.D. Macpherson. Omega-categoricity, relative categoricity and coordinatisation. *Annals of Pure and Applied Logic*, 46(4):169–199, 1990.
- [138] E. Hoogland. *Definability and interpolation. Model-theoretic investigations*. PhD thesis, Amsterdam, 2001. University of Amsterdam, ILLC Dissertation Series DS-2001-05.
- [139] P. Hraskó. Introduction to general relativity. Technical report, Technical University, Budapest, 1997. (in Hungarian).
- [140] W.v. Ignatowsky. *Phys. Zeitsch.*, 11:972, 1910.
- [141] G. James and R.C. James. *Mathematical Dictionary*. Van Nostrand, 1942.
- [142] G. Janelidze. Galois theory in categories: the new example of differentiable fields. In Adámek and Mac Lane [3], pages 369–380.
- [143] A. Jánossy, Á. Kurucz, and Á.E. Eiben. Combining algebraizable logics. *Notre Dame J. of Formal Logic*, 37(2):366–380, 1996.
- [144] P. Jipsen, B. Jónsson, and J. Rafter. Adjoining units to residuated Boolean algebras. *Algebra Universalis*, 34:118–127, 1995.
- [145] S.A. Jones, A. Morris and K.R. Pearson. *Abstract algebra and famous impossibilities*. Universitext, Springer-Verlag, Berlin, 1991.
- [146] B. Jónsson. Lattice-theoretic approach to projective and affine geometry. In Henkin et al. [131].
- [147] B. Jónsson. The preservation theorem for canonical extensions of Boolean algebras with operators. In K.A. Baker and Wille R., editors, *Lattice theory and its applications*, pages 121–130. Heldermann Verlag, 1995.
- [148] B. Jónsson and A. Tarski. Boolean algebras with operators. In *Alfred Tarski Collected Papers Vol. 3*, pages 369–420. Birkhäuser Verlag, 1986.
- [149] B. Jónsson and C. Tsınakis. Relation algebras as residuated Boolean algebras. *Algebra Universalis*, 30:469–478, 1993.

- [150] A. Joyal and R. Street. An introduction to Tannaka duality and quantum groups. In *Category theory (Como, 1990)*, volume 1488 of *Lecture Notes in Mathematics*, pages 413–492. Springer–Verlag, Berlin, 1991.
- [151] I. Kant. *Kritik der reinen Vernunft*. Johann Friedrich Hardknoch, Riga, 1781. Second revised edition: Druck und Verlag von Philipp Reclam jun, Leipzig, 1787. Later edition: Akademie Verlag (Kant’s Schriften), Berlin, 1910.
- [152] I. Kant. *Critique of Pure Reason*, volume XLII, 48 of *Great Books of the Western World*. Encyclopaedia Britannica Inc., Chicago, London, Toronto, 1952. Original German edition is [151].
- [153] C. Kassel. *Quantum Groups*. Springer-Verlag, New York, 1995.
- [154] A. Kirillov. *Elements of the theory of representations*. Springer–Verlag, Berlin, 1976.
- [155] A.I. Kostrikin and Yu.I. Manin. *Linear Algebra and Geometry*. Gordon and Breach, New York, 1988.
- [156] Á. Kurucz. *Decision problems in algebraic logic*. PhD thesis, Math. Inst. Hungar. Acad. Sci., Budapest, 1997. available from [www.math-inst.hu/pub/algebraic-logic/kuagthes.ps](http://www.math-inst.hu/pub/algebraic-logic/kuagthes.ps).
- [157] A. Kurusa. *Introduction to differential geometry (in Hungarian)*. Polygon (Bolyai Institute of University of Szeged), Hungary, 1999.
- [158] L.D. Landau and E.M. Lifsic. *Theoretical Physics Part II (theory of fields)*. Nauka, Moscow, 1973. in Russian.
- [159] R.W. Latzer. Non-directed light signals and the structure of time. In P. Suppes, editor, *Space, Time, and Geometry*. D.Reidel, 1973.
- [160] F.W. Lawvere. Adjointness in foundations. *Dialectica*, 23(3-4):281–296, 1969.
- [161] F.W. Lawvere. Metric spaces, generalized logic, and closed categories. *Rendiconti Seminario Matematico e Fisico di Milano* 43, 1973.
- [162] F.W. Lawvere. Variable quantities and variable structures in topoi. In *Algebra, topology, and category theory. A collection of papers in honor of Samuel Eilenberg*, pages 101–131. Academic Press, New York, 1976.
- [163] F.W. Lawvere and S.H. Schanuel, editors. *Categories in continuum physics*, volume 1174 of *Lecture Notes in Mathematics*. Springer–Verlag, Berlin, 1986.

- [164] H. Lenz. Fastgeordnete Körper. *Arch. Math.*, 11:333–338, 1960.
- [165] L. Lipshitz. The undecidability of the word problems for projective geometries and modular lattices. *Transactions of the American Mathematical Society*, 193:171–180, 1974.
- [166] J. Loose. *A historical introduction to the Philosophy of science*. Oxford Univ. Press, 1980.
- [167] H. Lugowski. *Foundations of universal algebra*. Teubner-Verlag, Leipzig, 1976. In German.
- [168] S. Mac Lane. *Categories for the working mathematician*. Springer-Verlag, Berlin, 1971.
- [169] J.X. Madarász. Epimorphisms in discriminator varieties. In *Proc. of Workshop on Abstract Algebraic Logic*, 1998. Centre de Recerca Matemàtica, Bellaterra (Spain), Quaderns nùm 10/ gener 1998, pp. 106-108.
- [170] J.X. Madarász. Interpolation and amalgamation; pushing the limits, Part I. *Studia Logica*, 61(3):311–345, 1998.
- [171] J.X. Madarász. Interpolation in algebraizable logics; semantics for non-normal multi-modal logic. *Journal of Applied Non-Classical Logics*, 8(1-2):67–105, 1998.
- [172] J.X. Madarász. On faster than light observers, Part II. Manuscript, Budapest, 1998.
- [173] J.X. Madarász. Surjectivity of epimorphisms in varieties of algebraic logic. Preprint, Alfréd Rényi Institute of Mathematics, Budapest, 2000.
- [174] J.X. Madarász. The symmetry axioms are equivalent in Basax, 2000.
- [175] J.X. Madarász and I. Németi. On faster than light observers, Part I. Manuscript, Budapest, 1998.
- [176] J.X. Madarász and I. Németi. FTL observers are at most two-dimensional. Manuscript in preparation, 1999.
- [177] J.X. Madarász and I. Németi. On FTL observers in a generalized version of first-order relativity (The generalization is in the direction of partiality coming from general relativity). Manuscript in preparation, 1999.

- [178] J.X. Madarász and T. Sayed-Ahmed. Amalgamation, interpolation, epimorphisms: solutions of all problems from Pigozzi's paper. Manuscript, Alfréd Rényi Institute of Mathematics, Budapest, 2001.
- [179] M. Makkai. Stone duality for first-order logic. *Adv. in Math.*, 65:97–170, 1987.
- [180] M. Makkai. Towards a categorical foundation of mathematics. In *Proc. Logic Coll*, Haifa, 1995.
- [181] M. Makkai. The structuralist program in mathematics. Preprint, McGill Univ. Canada, 1998. (also available: makkai@triples.math.mcgill.ca).
- [182] J. Makowsky and I. Sain. On the equivalence of weak second order and non-standard time semantics for various program verification systems. In *Logic in Computer Science (Proc. Conf. Cambridge USA 1986)*, 1986. IEEE Computer Society Press; pp.293-300.
- [183] K.L. Manders. First-order logical systems and set-theoretic definability. Preprint, University of Pittsburgh, 1983.
- [184] E.G. Manes. *Algebraic theories*, volume 26 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, Heidelberg, Berlin, 1976.
- [185] M. Manzano. Introduction to many-sorted logic. In Meinke and Tucker [193], pages 3–89.
- [186] N. Marti-Oliet and J. Meseguer. General logics and logical frameworks. In Gabbay [94], pages 355–392.
- [187] M. Marx. *Algebraic Relativization and Arrow Logic*. PhD thesis, University of Amsterdam, appeared as ILLC Dissertation Series 1995–3. ILLC, Plantage Muidergracht 24, 1018 TV Amsterdam, e-mail: ille@fwi.uva.nl, 1995.
- [188] M. Marx, L. Pólos, and M. Masuch, editors. *Arrow Logic and Multimodal Logic*. CSLI Publications, Stanford, 1996.
- [189] M. Marx and Y. Venema. *Multi-dimensional Modal Logic*. Kluwer Academic Press, 1997.
- [190] T. Matolcsi. *Spacetime without reference frames*. Akadémia Kiadó, Budapest, 1993.

- [191] T. Matolcsi. Spacetime without reference frames: an application to synchronizations on a rotating disk. Eötvös Loránd University, Budapest, preprint, submitted, 1999.
- [192] R.N. McKenzie, G.F. McNulty, and W.F. Taylor. *Algebras, Lattices, Varieties, Vol I*. The Wadsworth and Brooks/Cole Mathematics Series. Monterey, California, 1987.
- [193] K. Meinke and J.V. Tucker, editors. *Many-sorted Logic and its Applications*. John Wiley & Sons Ltd, 1993.
- [194] Sz. Mikulás. Conjugated arrow logic. Preprint, Math. Inst. of the Hung. Acad. of Sci., Budapest, 1992.
- [195] Sz. Mikulás. *Taming Logics*. PhD thesis, University of Amsterdam, appeared as ILLC Dissertation Series 1995–12. ILLC, Plantage Muidergracht 24, 1018 TV Amsterdam, e-mail: illc@fwi.uva.nl, 1995.
- [196] C.W. Misner, K.S. Thorne, and J.A. Wheeler. *Gravitation*. W. H. Freeman, San Francisco, 1973.
- [197] J.D. Monk. *Mathematical Logic*. Springer Verlag, 1976.
- [198] R. Montague. Deterministic theories. In *Decisions, Value and Groups, 2*, 1962. Oxford, Pergamon Press; pp.325-370.
- [199] J. Mundy. The physical content of Minkowski geometry. *Britisch J. Philos. Sci.*, 37(1):25–54, 1986.
- [200] D. Myers. An interpretive isomorphism between binary and ternary relations. In *Structures in logic and computer science*, 1997. ed. Jan Mycielski, Grzegorz Rozenberg, and Arto Salomaa, Lecture Notes in Computer Science 1261, Springer, Berlin 1997, pp.84-105.
- [201] G.L. Naber. *Spacetime and singularities, an introduction*. Cambridge Univ. Press, 1988.
- [202] L.D. Nel. Categorical differential calculus and Banach-Steinhaus. In Adámek and Mac Lane [3], pages 149–162.
- [203] I. Németi. Foundations for stepwise refinement of program specifications via cylindric algebra theory. *Diagrammes* (Paris), 8:1–24, 1982.

- [204] I. Németi. *Free Algebras and Decidability in Algebraic Logic*. Dissertation for D.Sc. with Hung. Academy of Sciences, Budapest, 1986. In Hungarian.
- [205] I. Németi. On varieties of cylindric algebras with applications to logic. *Annals of Pure and Applied Logic*, 36:235–277, 1987.
- [206] I. Németi. Algebraization of quantifier logics, an introductory overview. available from [www.math-inst.hu/pub/algebraic-logic/survey.ps](http://www.math-inst.hu/pub/algebraic-logic/survey.ps), 1997. (A shorter, incomplete version of this appeared in *Studia Logica*, 50(3/4)[a special issue devoted to Algebraic Logic, eds.: W.J. Blok and D.L. Pigozzi]:485–570, 1991).
- [207] I. Németi and I. Sain, editors. *Special issue of Logic Journal of IGPL on Algebraic Logic*. *Logic J. of IGPL* 8(4), 2000.
- [208] B. O’Neil. *The geometry of Kerr black holes*. A. K. Peters, 1995. ISBN: 1-56881-019-9.
- [209] I. Ozsváth and E.L. Schücking. The finite rotating universe. *Annals of Physics*, 55:166–204, 1969.
- [210] L.A. Pars. *Philos. Mag.*, 42:249, 1921.
- [211] J.W. Pelletier and J. Rosický. Generating the equational theory of  $C^*$ -algebras and related categories. In Adámek and Mac Lane [3], pages 163–180.
- [212] R. Penrose. *The Emperors’s New Mind*. Oxford University Press, 1989.
- [213] R. Penrose. *Shadows of the mind*. Oxford University Press, Oxford, New-York, 1994.
- [214] A. Pillay and S. Shelah. Classification theory over a predicate I. *Notre Dame Journal of Formal Logic*, 26(4):361–376, 1985.
- [215] M.B. Pour-El and J.I Richards. *Computability in analysis and physics*. Perspectives in mathematical logic. Springer–Verlag, Berlin, 1987.
- [216] S. Pulmanová. Quantum measurements and quantum logics. *Journal of Mathematical Physics*, 35:1555–1572, April, 1994.
- [217] W.V.O. Quine. *Philosophy of Logic*. Prentice-Hall, Englewood Cliffs, NJ., 1970.
- [218] M. Rédei. *Introduction to Quantum Logic*. Eötvös University Press, Budapest, 1995.

- [219] M. Rédei. *Quantum Logic in Algebraic Approach*, volume 91 of *Fundamental Theories of Physics*. Kluwer Academic Publishers, Dordrecht, Boston and London, 1998. pp. x+238.
- [220] M. Reed and B. Simon. *Methods of modern mathematical physics, Volumes I–IV*. Academic Press, New York, 1975.
- [221] T. Regge. *Infinito - Viaggi ai limiti dell’universo*. Mondadori, Milano, 1994.
- [222] H. Reichenbach. *The theory of relativity and a priori knowledge*. University of California Press, 1960. Translated by M. Reichenbach. (Original German edition published in 1920).
- [223] H. Reichenbach. *Axiomatization of the theory of relativity*. University of California Press, Berkeley, 1969. Translated by M. Reichenbach. Original German edition published in 1924.
- [224] W. Rindler. *Essential relativity. Special, general and cosmological*. Springer-Verlag, Berlin, 1969, 1977.
- [225] A.A. Robb. *A Theory of Time and Space*. Cambridge University Press, 1914. Revised edition, *Geometry of Time and Space*, published in 1936.
- [226] F. Rohlich. *From paradox to reality (our basic concepts of the physical world)*. Cambridge University Press, 1987.
- [227] H.L. Royden. Remarks on primitive notions for elementary Euclidean and non-Euclidean plane geometry. In Henkin et al. [131], pages 86–96.
- [228] P. Rózsa. *Linear algebra and its applications*. Tankönyvkiadó, Budapest, 1991. (in Hungarian).
- [229] B. Russel. *The problems of philosophy*. Williams and Norgate, London, 1990.
- [230] G. Sági. Lecture notes for logic and relativity. Manuscript, 1998.
- [231] I. Sain. *Dynamic logic with nonstandard model theory*. PhD thesis, Hungarian Academy of Sciences, Budapest, 1986. In Hungarian.
- [232] I. Sain. There are general rules for specifying semantics: observations on abstract model theory. *CL&CL*, 13:195–250, 1990.
- [233] W.C. Salmon. The philosophical significance of the one-way speed of light. *NOÛS*, 11:253–292, 1977.

- [234] S. Sankaran. Hochschild-Tannaka duality theorem for homogeneous spaces. *Rend. Sem. Mat. Univ. Politec. Torino*, 36:59–85, 1979.
- [235] P. Schauenburg. *Tannaka duality for arbitrary Hopf algebras*, volume 66 of *Algebra Berichte*. Verlag Reinhard Fischer, Munich, 1992. ii+57 pp.
- [236] J.W. Schutz. *Independent axioms for Minkowski space-time*. Longoman, London, 1997.
- [237] W. Schwabhäuser, W. Szmielew, and A. Tarski. *Metamathematische Methoden in der Geometrie*. Hochschul text. Springer-Verlag, Berlin, 1983.
- [238] I.R. Shafarevich. *Basic Notions of Algebra*, volume 11 of *Encyclopedia of Mathematical Sciences*. Springer-Verlag, Berlin, 1997.
- [239] S. Shelah. *Around classification theory of models*. Lecture Notes in Mathematics 1182, Springer-Verlag, Berlin, 1986.
- [240] H. Stein. On relativity theory and openness of the future. *Philosophy of Science*, 58:147–167, 1991.
- [241] W.F. Stinespring. Integration theorems for gauges and duality for unimodular groups. *Trans. Amer. Math. Soc.*, 90:15–56, 1959.
- [242] M. Stöltzner. Gödel and the theory of everything. In Hájek [118], pages 291–306.
- [243] P. Suppes. The axiomatic method in the empirical sciences. In L. Henkin, J. Addison, C.C. Chang, W. Craig, D. Scott, and R. Vaught, editors, *Proceedings of the Tarski Symposium*, Proceedings of Symposia in Pure Mathematics Volume XXV, pages 465–479, Providence, Rhode Island, 1974. American Mathematical Society.
- [244] L.E. Szabó. The problem of open future, determinism in the light of relativity and quantum theory. Manuscript, Dept. of Theor. Physics, Eötvös Loránd University, Budapest, 1998.
- [245] L.W. Szczerba and A. Tarski. Metamathematical properties of some affine geometries. In Y. Bar-Hiller, editor, *Proceedings of the 1964 International Congress for Logic, Methodology and Philosophy of Science*, Studies in Logic and the Foundations of Mathematics, pages 166–178, 1965.

- [246] W. Szmielew. The role of the Pasch axiom in the foundations of Euclidean geometry. In *Proc. of the Tarski Symp. held in Berkeley in 1971*, pages 123–132. Amer. Math. Soc., Providence, RI, 1974.
- [247] W. Szmielew. *From Affine to Euclidean Geometry*. Reidel Publ. Co., 1983.
- [248] A. Tarski. A problem concerning the notion of definability. In [252], Vol. 3, pp. 163–170.
- [249] A. Tarski. Some methodological investigations on the definability of concepts. (in German), in [252], Voll 1, pp.637–639. The Polish version contains more detail: in *Revue Philosophique*, Vol.37 (1934).
- [250] A. Tarski. Fundamentale begriffe der methodologie der deduktiven wissenschaften, i. *Monatsh. Math. Phys.*, 37:361–404, 1930. Also in “Logic, Semantics, Metamathematics. Papers from 1923 to 1938. Clarendon Press, Oxford, 1956. pp.60–109.
- [251] A. Tarski. What is elementary geometry? In Henkin et al. [131], pages 16–29.
- [252] A. Tarski. *Collected Papers, Volumes 1–4*. Birkhäuser (Basel–Boston–Stuttgart), 1986. Editors: Steven R. Givant and Ralph N. McKenzie.
- [253] A. Tarski and S. Givant. *A Formalization of Set Theory Without Variables*, volume 41 of *AMS Colloquium Publications*. Providence, Rhode Island, 1987.
- [254] A. Tarski and S. Givant. Tarski’s system of geometry. *Bulletin of Symbolic Logic*, 5,2:175–214, 1999.
- [255] A. Tarski, A. Mostowski and R.M. Robinson. *Undecidable theories*. North-Holland, Amsterdam, 1953. xii+98pp.
- [256] E.F. Taylor and J.A. Wheeler. *Spacetime Physics, second edition, fourth printing*. W. H. Freeman and Company, New York, 1997.
- [257] R.J. Thompson. Complete description of substitutions in cylindric algebras and other algebraic logics. In *Algebraic Methods in Logic and in Computer Science* (Proceedings of the ’91 Banach Semester, Banach Center), pages 327–342, Warsaw, 1993. Polish Academic Publishers, Banach Center Publications, Volume 28.
- [258] K.S. Thorne. *Black holes and time warps*. W. W. Norton and Company, 1994.

- [259] Cs. Tóke. Some problems in the analysis of relativity theory in first-order logic. Master's thesis, Eötvös Loránd University, Budapest, 2000. 58pp.
- [260] A. Urquhart. Decision problems for distributive lattice-ordered semigroups. *Algebra Universalis*, 34:399–418, 1995.
- [261] F. Valentine. *Convex sets*. McGraw Hill, 1964.
- [262] S. Vályi. Modal version of basic axiom system for special relativity. Manuscript, Institute of Mathematics, Kossuth Lajos University Debrecen, 1998.
- [263] J. van Benthem. *Modal logic and classical logic*. Bibliopolis, Napoli, 1983.
- [264] J. van Benthem and A. ter Meulen (eds). *Handbook of Logic and Language*. North-Holland, Amsterdam, 1997.
- [265] J.A.F.K. van Benthem. *The Logic of Time*. D. Reidel, 1983.
- [266] J.A.F.K. van Benthem. *Language in Action*. Elsevier Science Publishers, Amsterdam, 1991.
- [267] J.A.F.K. van Benthem. Content versus wrapping: an essay in semantic complexity. In Marx et al. [188], pages 203–219.
- [268] J.A.F.K. van Benthem. *Exploring logical dynamics*. Studies in Logic, Language and Information. CSLI Publications, Stanford, 1996.
- [269] D. van Dalen. *Logic and Structure*. Springer Universitext, 1997. Third Edition, Second printing.
- [270] V.S. Varadarajan. *Geometry of Quantum Theory*, volume I. D. Van Nostrand Co. Inc., Princeton, New Jersey, 1968.
- [271] Y. Venema. *Many-dimensional Modal Logic*. PhD thesis, University of Amsterdam, 1992.
- [272] J. von Neumann. *Continuous Geometry*, volume 25 of *Princeton Mathematical Series (I. Halperin, ed.)*. Princeton University Press, Princeton NJ, 1960. xi+299 pp.
- [273] K.G.C. von Staudt. *Beiträge zur Geometrie der Lage*. F. Korn, Nürnberg, 1857.

- [274] R.M. Wald. *General relativity*. The University of Chicago Press, Chicago, 1984.
- [275] J.A. Winnie. Special relativity without one-way velocity assumptions. *Philosophy of Science*, 37:81–99; 223–238, 1970.
- [276] E.C. Zeeman. Causality implies the Lorentz group. *Journal of Mathematical Physics*, 5:490–493, 1964.
- [277] P. Zlatos. On conceptual completeness of syntactical-semantical systems. *Periodica Math. Hungar.*, 16(3):145–174, 1985.

Alfréd Rényi Institute of Mathematics  
Budapest, Pf. 127  
H-1364, Hungary  
e-mail: andreka.madarasz.nemeti@renyi.hu