The Counting of Crossing-Free Geometric Graphs — Algorithms and Combinatorics

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Abstract

We are interested in the understanding of crossing-free geometric graphs–these are graphs with an embedding on a given planar point set where the edges are drawn as straight line segments without crossings. Often we are restricted to certain types of graphs, most prominently triangulations, but also spanning cycles, spanning trees, or (perfect) matchings (and crossing-free partitions), among others. A primary goal is to enumerate, to count, or to sample graphs of a certain type for a given point set–so these are algorithmic questions–, or to give estimates for the maximum and minimum number of such graphs on any set of n points–these are problems in extremal combinatorial geometry.

Among others, I will show some of the new ideas for providing extremal estimates, e.g. for the number of crossing-free spanning cycles: the support-refined estimate for cycles versus triangulations, the use of pseudo-simultaneously flippable edges in triangulations, and the employment of Kasteleyn's beautiful linear algebra method for counting perfect matchings in planar graphs–here, interestingly, in a weighted version. Moreover, Alvarez and Seidel's recent 2^n-algorithm for counting triangulations is discussed, with Wettstein's extensions to other types of graphs (e.g. crossing-free perfect matchings). This allows the first efficient enumeration algorithm (i.e. with polynomial delay) for crossing-free perfect matchings.

Keywords: computational geometry, geometric graphs, counting, sampling, enumeration