

Middle-Level Graphs

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In this note we examine the class of Middle-Level Graphs, $ML(k)$, also known as the Revolving Door Graphs. $ML(k)$ is defined as the subgraph of the $(2k + 1)$ -dimensional cube, $Q(2k + 1)$, induced by the vertices with either exactly $k + 1$ ones or exactly k ones. These graphs have been studied extensively in an attempt to settle the conjecture that they are Hamiltonian. They are known to be distance-transitive and therefore distance-regular. We will prove some results about their embedding in the cube and examine the middle level of the middle level. In particular we show that the middle level of $ML(k)$ consists of disconnected copies of middle level graphs of lower dimension. Thus we show that the middle level of $ML(k)$ is the join of $\binom{k}{k/2}$ copies of $ML(k/2)$ when k is even and the join of $\binom{k+1}{(k+1)/2}$ copies of $ML((k - 1)/2)$ when k is odd.