Decomposition of multiple packings with subquadratic union complexity

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(joint work with János Pach)

Let k be a positive integer and $\mathcal{X}$ be a *k-fold packing* of simply connected compact sets in the plane, that is, a family such that every point belongs to at most k sets. Suppose that there is a function $f(n) = o(n^2)$ with the property that any n members of $\mathcal{X}$ surround at most $f(n)$ *holes*, which means that the complement of their union has at most $f(n)$ bounded connected components. We use tools from extremal graph theory and the topological Helly theorem to prove that $\mathcal{X}$ can be decomposed into at most p packings, where p is a constant depending only on k and f.

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