Making a C_6-free graph C_4-free and bipartite

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Let $e(G)$ denote the number of edges in a graph G, and let C_k denote a k-cycle. It is well-known that every graph has a bipartite subgraph with at least half as many edges. Győri showed that any bipartite, C_6-free graph contains a C_4-free subgraph containing at least half as many edges. Applying these two results in sequence we see that every C_6-free graph, G, has a bipartite C_4-free subgraph, H, with $e(H) \geq e(G)/4$. We show that the factor of $1/4$ can be improved to $3/8$:

Theorem 1. *Let G be a C_6-free graph, then G contains a subgraph with at least $3e(G)/8$ edges which is both C_4-free and bipartite.*

The proof uses probabilistic ideas combined with a charactarization of C_6-free graphs due to Füredi, Naor and Verstraëte.