Counting double-normal pairs

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(joint work with János Pach)

Given a set of n points, there are various ways of declaring two points to be "far apart". Two well-known such notions are diameter pairs, where the distance between the points equals the diameter of the set (first considered by Erds), or antipodal pairs (introduced by Klee), where there exist parallel hyperplanes through the two points with the whole set contained in the closed slab bounded by the hyperplanes. Martini and Soltan (2005) introduced the notion of a double-normal pair, where we ask in addition to antipodality that the parallel hyperplanes are perpendicular to the line joining the two points. This notion interpolates between that of diameter pairs and antipodal pairs.

In this talk we discuss the problems of estimating the maximum number of diameters, antipodal pairs, or double-normal pairs in a set of n points in Euclidean space. The problems for diameters and antipodal pairs are well known, but nothing has previously been done for double-normal pairs.