Hypergraphs and Geometry

Noga Alon, Tel Aviv University

All four birthday boys have obtained beautiful results combining geometric tools and ideas with extremal questions on graphs and hypergraph. After a brief discussion of some of these results I will describe a recent application of properties of high dimensional Euclidean spaces in the study of an extremal hypergraph problem extending a question of Erdős and Rothschild.

Ramsey numbers of ordered graphs

Martin Balko

(joint work with Karel Kral and Jan Kyncl)

An ordered graph $G_<$ is a graph G with vertices ordered by the linear ordering $<$. The ordered Ramsey number $R(G_<, c)$ is the minimum number N such that every ordered complete graph with c-colored edges and at least N vertices contains a monochromatic copy of $G_<$.

For unordered graphs it is known that Ramsey numbers of graphs with degrees bounded by a constant are linear with respect to the number of vertices. In contrast with this result we show that there are arbitrarily large ordered matchings $M_<(n)$ on n vertices for which $R(M_<(n), 2)$ grows super-polynomially in n. This implies that ordered Ramsey numbers of the same graph can grow super-polynomially in the size of the graph in one ordering and remain polynomial in another ordering.

We also prove that for every ordered graph its ordered Ramsey number grows either polynomially or exponentially in the number of colors.

For a few special classes of ordered paths, stars or matchings, we give asymptotically tight bounds on their ordered Ramsey numbers. For so-called monotone cycles we compute their ordered Ramsey numbers exactly. This result implies exact formulas for geometric Ramsey numbers of cycles introduced by Károlyi et al.

On the typical structure of sum-free sets.

József Balogh, Szeged University and UIUC

(Based on joint work results with Alon, Morris, Samotij and Warnke)

First we study sum-free subsets of the set $\{1,\ldots,n\}$, that is, subsets of the first n positive integers which contain no solution to the equation $x+y=z$. Cameron and Erdős conjectured in 1990 that the number of such sets is $O(2^{n/2})$. This conjecture was confirmed by Green and, independently, by Sapozhenko. We prove a refined version of their theorem, by showing that the number of sum-free subsets of $[n]$ of size m is $2^{O(n/m)}\binom{\lceil n/2\rceil}{m}$, for every $1 \le m \le \lceil n/2\rceil$. For $m \ge \sqrt{n}$, this result is sharp up to the constant implicit in the $O(\cdot)$. Our proof uses a general bound on the number of independent sets of size m in 3-uniform hypergraphs, proved recently by the authors, and new bounds on the number of integer partitions with small sumset.

Then we study sum-free sets of order m in finite Abelian groups. We determine the typical structure and asymptotic number of sum-free sets of order m in Abelian groups G whose order n is divisible by a prime q with $q \equiv 2 \pmod 3$, for every $m \ge C(q)\sqrt{n\log n}$, thus extending and refining a theorem of Green and Ruzsa. In particular, we prove that almost all sum-free subsets of size m are contained in a maximum-size sum-free subset of G.

Finally, we explain connection with recent "independent sets in hypergraph" general theorems, and describing typical structure of graphs.

In the talk I try to have different approach from other talks on "independent sets in hypergraph" general theorems.

Partition regularity and the columns property

Ben Barber, University of Birmingham

A system of linear equations with integer coefficients is partition regular if, whenever the natural numbers are finitely coloured, there is a monochromatic solution. In 1933 Rado showed that a finite system of equations is partition regular if and only if its matrix of coefficients has the "columns property".

It is easy to write down infinite systems which have the columns property but are not partition regular. However, all known examples of infinite partition regular systems do have the columns property. Must all infinite partition regular systems have the columns property?

EDGE-COLORINGS OF GRAPHS AVOIDING COMPLETE GRAPHS WITH A PRESCRIBED COLORING PATTERN

FABRÍCIO SIQUEIRA BENEVIDES, CARLOS HOPPEN, AND RUDINI MENEZES SAMPAIO

Abstract. For any fixed graph F, we say that a graph G is F-free if it does not contain F as a subgraph. We denote by $\mathrm{ex}(n, F)$ the maximum number of edges in a n-vertex graph which is F-free, known as the Turán number of F.

In 1974, Erdős and Rothschild considered a related question where we count the number of certain colorings. Given an integer r, by an r-coloring of a graph G we mean any r-edge-coloring of G. In particular, it does not have to be proper and does not have to use all r colors. Let $c_{r,F}(G)$ be the number of r-colorings of G such that *every color class is F-free.* They considered the problem of finding $c_{r,F}(n) = \max\{c_{r,F}(G)\}$ where the maximum is over all n-vertex graphs G. Let us say that G is extremal for $c_{n,F}(n)$ if it realizes the above maximum. Clearly, $c_{r,F}(n) \geq r^{\mathrm{ex}(n,F)}$, as we take G to be the Turán graph and color it arbitrarily. The problem of determining $c_{r,F}(n)$ was investigates by several authors, for various classes of graphs such as: complete graphs [1, 8, 9], odd cycles [1], matchings [4], paths and stars [5]. And for hypergraphs [3, 6, 7]. One common concern is to determine when the Turán Graph is extremal for $c_{r,F}(n)$ (with r fixed and n large).

Here we consider a natural generalization of the above. Given an r-colored k-vertex graph $\hat{F}$, we consider the number of r-edge-colorings of a larger graph G that avoids the 'color pattern' of $\hat{F}$. More formally, $c_{r,\hat{F}}(G)$ denote the number or r-colorings of G such there are no k vertices of G that induce a colored graph isomorphic to $\hat{F}$. For example, the above problem consists of the case where $\hat{F}$ is a colouring of F that uses only one of the r colors. We define $c_{r,\hat{F}}(n)$ and extremal graphs as before.

We note that Balogh [2] had also considered a related but not analogous "colored version" of the problem. He considered the number $C_{r,\hat{F}}(G)$ of colorings of G which do not have a set of k-vertices colored exactly as in $\hat{F}$. In this case, for example, if $\hat{F}$ has only one color, $C_{r,H}(G)$ is the number of coloring of G which does not contains $\hat{F}$ in this particular color class.So $c_{r,\hat{F}}(G) \leq C_{r,\hat{F}}(G)$. Balogh proved that in the case where $r = 2$ and $\hat{F}$ is a 2-coloring of a clique that uses both colors then $C_{2,\hat{F}}(n) = 2^{\mathrm{ex}(n,\hat{F})}$ for n large enough.

Here, we focus on the case where $r = 3$. Let $\hat{F}_3$ be a 3-colored K_3. We proved that if the three colors are used in $\hat{F}_3$ then the complete graph on n vertices is the extremal graph for $c_{3,\hat{F}_3}(n)$. And if only two colors are used in F_3 then the Turán Graph is extremal for $c_{3,\hat{F}_3}(n)$ (whereas this is trivially not true for $C_{3,\hat{F}_3}(n)$). Much more generally we prove the following: with $r = 3$, let $\hat{F}_k$ be a coloring of K_k that uses only two colors one of which induces a graph H whose Ramsey Number is smaller than k, then the Turán Graph is extremal for $c_{3,\hat{F}_k}(n)$.

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Some developments of the weighted EGZ theorem

Arie Bialostocki

(joint work with Matthew Conroy)

It has been conjectured that for n even if A and B are two zero sum sequences, over Z_n, each of length n, then there exists a permutation which permutes the elements of B resulting a sequence B' such that the inner product of A and B' is 0 in Z_n. First, we extend the above conjecture to n odd, provided A and B do not belong to two exceptional cases. Next, we provide some information about the exceptional cases and other cases of interest.

Large standard examples in posets of high dimension

Csaba Bíró, Louisville University

(joint work with Péter Hamburger and Attila Pór)

A classic theorem by Hiraguchi states that the dimension of a partially ordered set of n elements is not more than $n/2$. Bogart and Trotter proved that if the dimension is exactly $n/2$, then the poset is isomorphic to one specific poset called the "standard example". A natural question is the following: if the dimension is slightly less than $n/2$, is the poset largely similar to a standard example? We study several questions in the area, and we get positive and negative answers depending on the setting.

The Time of Bootstrap Percolation –
Extremal and Probabilistic Results

Béla Bollobás
Cambridge, Memphis and LIMS

Classical r-neighbour bootstrap percolation, introduced by Chalupa, Leath and Reich in 1979, can be viewed as an oversimplified model of the spread of an infection: given a graph G and a set A_0 of 'infected' vertices at time 0, for $t \geq 0$ we define

$$A_{t+1} = A_t \bigcup \big\{x \in V(G) : \ x \text{ has at least } r \text{ neighbours in } A_t\big\}$$

to be the set of infected vertices at time $t+1$. The set A_0 is said to *percolate* if its *closure*, $[A_0] = \bigcup_t A_t$, is the entire vertex set $V(G)$. The *percolation time* of A_0 is

$$T(A_0) = \min\{t : \ A_t = V(G)\}.$$

Most of the work in the last thirty years has been about what happens when the initial set A_0 is chosen at random, with major contributions by Aizenman, Balogh, Bollobás, Cerf, Cirillo, Duminil-Copin, Holroyd, Lebowitz, Manzo, Morris, and others.

In the first half of this talk I shall sketch the most significant of these results, and then I shall turn to some recent work of Balister, Benevides, Bollobás, Holmgren, Przykucki and Smith, emphasizing the extremal problems that arise.

The second part of the talk concerns *graph bootstrap percolation*, a rather different kind of bootstrap percolation I introduced in 1968 under another name, about which the first beautiful results were obtained by Frankl, Kalai and Alon in the 1980s. My aim is to say a little about a number of recent extremal and probabilistic results of Balogh, Bollobás, Koch, Morris and Przykucki.

Cross-intersecting families

Peter Borg

Department of Mathematics, University of Malta

Extremal set theory is the study of how small or how large a system of sets can be under certain conditions. A problem in this field that has recently attracted much attention is that of determining the maximum sum or the maximum product of sizes of $k \geq 2$ *cross-t-intersecting* subfamilies of a given family $\mathcal{F}$ of sets; families $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_k$ are said to be cross-t-intersecting if for every i and j in $\{1, 2, \ldots, k\}$ with $i \neq j$, each set in $\mathcal{A}_i$ intersects each set in $\mathcal{A}_j$ in at least t elements. Solutions have been obtained for various important families $\mathcal{F}$, such as power sets, levels of power sets, hereditary families, families of permutations, and families of integer sequences. The talk will provide an outline of these results, together with some general observations and results.

Minkowski valuations on lattice polytopes

Károly Böröczky, Central European University

(joint work with Monika Ludwig)

We characterize translation invariant, and either $SL(n, Z)$ equivariant or $SL(n, Z)$ contravariant Minkowski valuations on lattice polytopes.

Copies of fixed graphs in random distance graphs

Anton Burkin*

The problem of appearance of an arbitrary fixed graph in Erdős–Rényi random graph $G_{n,p}$ was studied thoroughly by a number of authors (see, e.g., [1]). For a certain model of random distance graphs this issue was investigated by M. Zhukovskii in [2]. We consider a class of distance graphs $G(n,r,s) = (V(n,r), E(n,r,s))$ defined as follows:

$$V(n,r) = \{\mathbf{x} = (x_1, \dots, x_n) \colon x_i \in \{0,1\},\ x_1 + \dots + x_n = r\}, \quad E(n,r,s) = \{\{\mathbf{x}, \mathbf{y}\} \colon (\mathbf{x}, \mathbf{y}) = s\},$$

where $(\mathbf{x}, \mathbf{y})$ is the Euclidean scalar product.

We study the random distance graphs $\mathcal{G}(G(n,r,s), p)$ where each edge from the set $E(n,r,s)$ is included in the graph with probability p independent of other edges. We find the threshold probabilities for the property of containing a fixed graph and investigate the distribution of the number of subgraphs isomorphic to a given graph when the probability p is critical. We also find the theshold probabilities for planarity in these graphs.

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Three Open Questions related to the Tick Data Decomposition Problem

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Keywords: combinatorial optimization, tick data, data storage

The tick data decomposition problem is a combinatorial optimization problem motivated by real-world applications, in particular, by the need for efficient storage structures for discrete-valued multivariate time-series, such as the data describing financial transactions, or bag of words vectors of dynamically changing texts such as blogs or Wikipedia pages. Here, we will describe the tick data decomposition problem and we will point out three open questions related to the tick data decomposition problem.

In many applications, various attributes of an object are measured continuously over time. A *tick data matrix* M is a matrix where columns correspond attributes or features while rows correspond observations of the same features at different time points. Rows of the matrix are ordered according to the order of observations, i.e., the values of the i-th row were observed *before* the values of the j-th row if and only if $i < j$. While the observations are made, a new row is added whenever the value of one or more attribute(s) change(s). However, as long as none of the attribute-values changes no new row is added to the matrix, therefore two rows of a tick data matrix differ in the value of at least one attribute. There is an additional column that is used to index the rows of a tick data matrix. This additional *index column* may contain, for example, ascending integer numbers (like the number of the corresponding row) or a time-stamp (see the *Time* column in the example shown in Figure 1). We use the term *regular column* for all the columns other than the index column.

With *decomposition* of a tick data matrix M we mean the partitioning of the regular columns of M into k disjoint partitions P_i, $1 \leq i \leq k$, i.e., for each regular column c_j of M: $c_j \in P_1 \vee c_j \in P_2 \vee ... \vee c_j \in P_k$; and for all i, j with $i \neq j$ $P_i \cap P_j = \emptyset$. Note that this partitioning refers to the regular columns only, i.e., in this formulation, the index column does not belong to any cluster. Then, for each cluster P_i, a matrix M_i is derived from M by selecting the index column and those columns of M that belong to cluster P_i. Subsequent rows of a derived matrix M_i may contain the same values in all the regular columns. In such cases we only keep the first row. For example, in Figure 1, $P_1 = \{$Humidity, Pressure$\}$, $P_2 = \{$Temperature, Wind (velocity), Wind (direction), Radiation, Outlook$\}$ and the corresponding matrices M_1 and M_2 are shown in the bottom left and bottom right of the Figure 1.

We can easily see that the original matrix can be reconstructed from the decomposition described above, and therefore, instead of the original matrix M, one can use this decomposition to calculate the results of search and analytic queries. Furthermore, as we have shown in our previous works, this decomposition allows to process queries *efficiently*, i.e., without the explicit need for decompressing the data, and simultaneously it leads to substantial improvements in terms of storage space [1, 2].

Consequently, we can state the tick data decomposition problem as follows.

Problem 1 *For a given number of clusters k, we aim at finding a decomposition so that the total number of the cells in all the matrices M_i is minimized.*

The above problem statement directly gives two variants of the tick data decomposition problem: while counting the number of cells in the matrices M_i, we can either count the cells in the index column or not.

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a)

Time	Temp. (°C)	Hum. (%)	Press. (Pa)	Wind (v) (km/h)	Wind (dir.)	Radiation	Outlook
10:21	15	20	100 200	5	SW	low	
10:22	16	20	100 200	5	SW	low	
10:38	16	30	100 100	5	SW	low	
10:40	17	30	100 100	5	SW	medium	
10:43	18	30	100 100	10	SW	medium	
10:44	18	30	100 100	15	W	medium	
10:51	18	20	100 200	15	W	medium	

b)

Time	Hum. (%)	Press. (Pa)
10:21	20	100 200
10:38	30	100 100
10:51	20	100 200

Time	Temp. (°C)	Wind (v) (km/h)	Wind (dir.)	Radiation	Outlook
10:21	15	5	SW	low	
10:22	16	5	SW	low	
10:40	17	5	SW	medium	
10:43	18	10	SW	medium	
10:44	18	15	W	medium	

Figure 1: An illustrative example for tick data. Features describing the weather are monitored continuously. Whenever the value of one of the features changes, a new row is inserted into the recordings (section a). Decomposition of such tables by features (columns) that change their values simultaneously may substantially reduce the required storage space (section b).

Furthermore, the above problem statement implicitly assumes uniform storage costs for all the cells, as it simply targets to minimize the number of cells in the decomposition. Other variants of the tick data decomposition problem may not assume uniform storage cost for each cells.

We note that k is usually relatively small: for example, for the storage of tick data of financial transactions, the user is most interested in the decomposition into $k = 2$ or $k = 3$ clusters. This is because, in case of real data, according to our observations, the decomposition into two or three partitions already leads to substantial gain in terms of storage space, and the decomposition into more partitions do leads to only minor further improvements, whereas the average computational costs of a query may grow with increasing k, see also [1].

In our previous work, we proposed an iterative, greedy algorithm for the tick data decomposition problem [2]. In the first iteration, this algorithm considers each column as a separate partition, then, in each iteration, it merges those two partitions that lead to optimal storage size. In [1], we gave a computationally cheap lower bound for the storage size in order to speed up the algorithm.

Despite its relevance from the point of view of applications, the theoretical foundations of the tick data decomposition problem are largely unclear and the authors are not aware of other combinatorial optimization problems that are equivalent to this problem. Therefore, in order to motivate discussions, we pose the following open questions related to the tick data decomposition problem:

1. Under which assumptions is it *possible* to find a good decomposition of a tick data table, i.e., a decomposition that leads to substantial improvements in terms of storage size?

2. What is the complexity of the tick problem? Depending on the assumptions about the data, are there cases in which the optimal decomposition is “simple” (or “difficult”) to find?

3. In which cases do simple greedy algorithms find the optimal, or close to optimal decompositions?

Similar questions were successfully studied in context of various optimization problems resulting in celebrated results such as the theorems related to bin packing or Kruskal’s algorithm for searching for the minimal spanning tree in graphs. We hope that the study of the above questions may contribute to establish the theoretical framework of the tick data decomposition problem.

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On property B of hypergraphs

D.D. Cherkashin [1]

I am going to speak about a classical quantity $m(n)$ introduced by Erdős and Hajnal in 1961 (see [1]).

A hypergraph $H = (V, E)$ is said to have *property* B, if there is a 2-coloring of V with no monochromatic edges. Denote by $m(n)$ the minimum number of edges in a hypergraph that does not have property B.

The best known bounds for $m(n)$ are as follows:

$$c\sqrt{\frac{n}{\ln n}}2^n < m(n) < c'n^2 2^n.$$

The lower bound is due to Radhakrishnan and Srinivasan (see [2]), and the upper bound was given by Erdős.

I want to present a new simple proof of the lower bound (based on ideas by A. Pluhár from [3]) and a new lower bound for a quantity $m(n, r)$ that generalizes $m(n)$ onto the case of r colors.

This is my joint work with J. Kozik.

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An efficient Extension for Sperner families:
based upon from m-cardination Sperners sets to m+1-cardination

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Abstract

Sperner family (or Sperner system) is a set system {S(E)}where S(E), the Sperner set with the generating set E, is a set in which no element is contained in another. Formally, S(E) is composed of the elements, i.e. Sperner elements, from power set of E and for any different *X*, *Y* in S(E), *X* is not contained in *Y* and *Y* is not contained in *X*. Great efforts have been made to compute or estimate the number N, the cardination of the{S(E)} with n, the cardination of E. N increases rapidly with n. For example, N=7579 for n=5 and N=7828352 for n=6. So far the largest N is known for n=7. Therefore, the construction of Sperner family for large n is still a changeling problem in present days.

It is noted that the Sperner family is the set of the all Sperner sets for a specified E. We therefore classify the Sperner family into different categories (sub-family) by the cardination of the Sperner set. Furthermore, we recursively construct the Sperner set with cardination increasing from m to m+1. A direct recursive computation from m to m+1 is to check the 2^n different 1-cardination Sperner elements for compatibilities with the interested m-cardination Sperner sets, which costs $2^n * m$ inclusion comparisons. Instead, we find the compatibility for a specific child Sperner element from its possibly r younger brothers with a same parent of m-1-cardination Sperner set. It is noted that the above specific child is corresponding to a specific Sperner set of m-cardination and thus reduces the inclusion comparisons from 2 n * m to r instead of C(r, 2), selecting 2 from r. The number r is small in general and m is from 1 to C(n, (n-1)/2) according to the Sperner theorem.

A proof of correctness for no missing Sperners set and no repeating one in extension by the proposed strategy is also discussed in this paper. An example diagram is also provided for illustrating the realization idea.

Keywords: Sperner family, Sperner set, Sperner element, recursive computation

An example diagram for illustrating the realization idea:

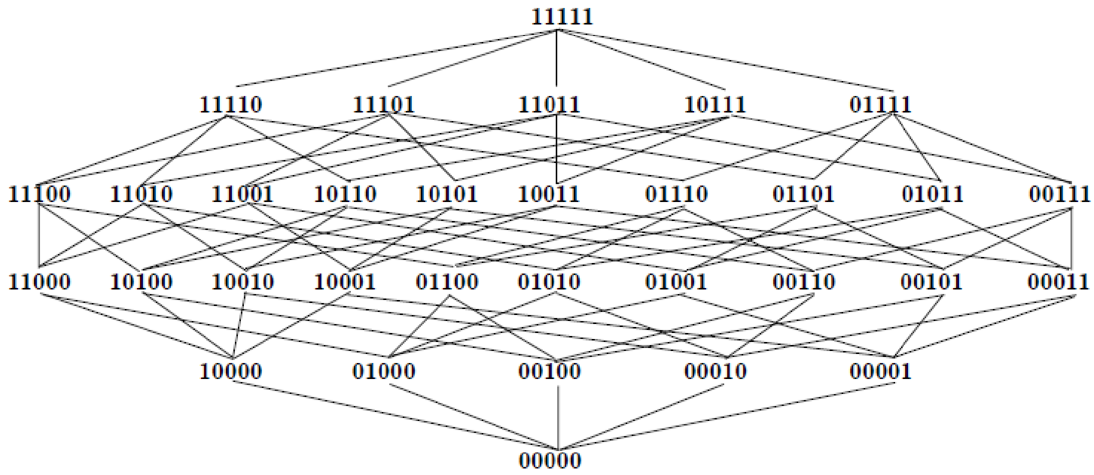

Fig. 1 Sperner elements (n=5) with their containing relationship represented in graph

14	13	11
14	13	7
14	13	3
14	11	7
14	11	5
14	9	7
14	9	5
14	9	3
14	5	3
13	11	7
13	11	6
13	10	7
13	10	6
13	10	3
13	6	3
12	11	7
12	11	6
12	11	5
12	10	9

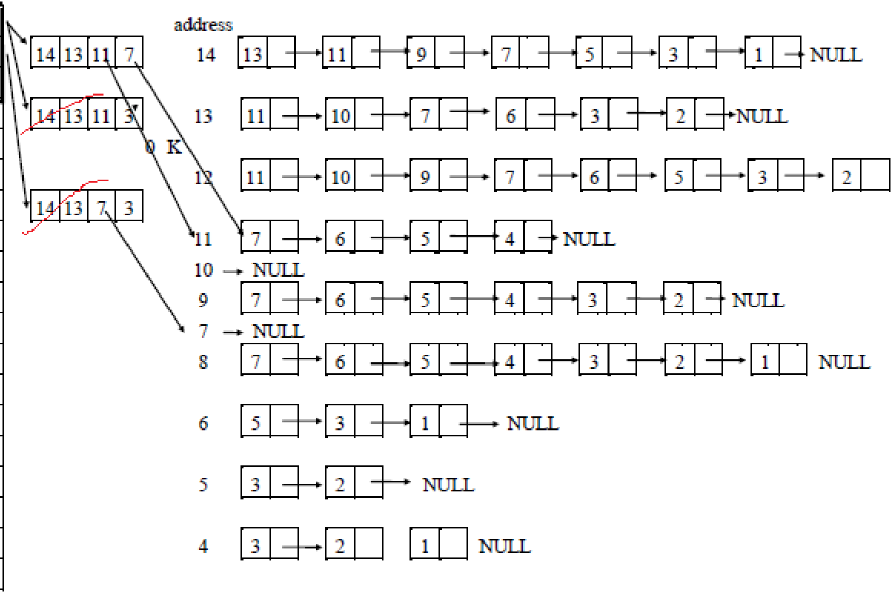

Fig. 2 One instance of Sperner set extension from 3-cardination to 4-cardination: The first extension is successful but the second and the third are not. (14 = 01110, …..)

Füredi–Hajnal constants are typically subexponential

Josef Cibulka and Jan Kynčl

Abstract

A *binary matrix* is a matrix with entries from the set $\{0, 1\}$. We say that a binary matrix A *contains* a binary matrix B if B can be obtained from A by removal of some rows, some columns, and changing some 1-entries to 0-entries. If A does not contain B, we say that A *avoids* B. A *permutation matrix* P is a binary square matrix with exactly one 1-entry in every row and one 1-entry in every column.

The Füredi–Hajnal conjecture, proved by Marcus and Tardos in 2004, states that for every permutation matrix P, there is a constant c_P such that for every $n \in \mathbb{N}$, every $n \times n$ binary matrix A with at least $c_P n$ 1-entries contains P. Klazar proved that the Füredi–Hajnal conjecture implies the Stanley–Wilf conjecture, which states the following. For every permutation matrix P, there is a constant s_P such that for every $n \in \mathbb{N}$, the number of $n \times n$ permutation matrices avoiding P is at most s_P^n.

Fox recently found a randomized construction showing that for every k, there are $k \times k$ permutation matrices P with $c_P \geq 2^{\Omega(k^{1/4})}$. He additionally showed that as k goes to infinity, almost all $k \times k$ permutation matrices satisfy $c_P \geq 2^{\Omega((k/\log(k))^{1/4})}$. Fox also improved the original Marcus–Tardos upper bound $c_P \leq 2k^4\binom{k^2}{k}$ to $c_P \leq 2^{8k}$ (which can be easily lowered to $c_P \leq 2^{6k}$), for all $k \times k$ permutation matrices P.

A 1-entry in a matrix is identified by the pair (i, j) of the row index i and the column index j. The *distance vector* between the entries (i_1, j_1) and (i_2, j_2) is $(i_2 - i_1, j_2 - j_1)$. We say that a $k \times k$ permutation matrix P is *scattered* if every pair (d, d') is the distance vector of at most $\log_2(k)$ pairs of 1-entries of P. As k goes to infinity, almost all $k \times k$ permutation matrices are scattered.

We show that $c_P \leq 2^{O(k^{2/3}\log^{7/3}(k))}$ for every scattered $k \times k$ permutation matrix P. The main part of the proof is showing that every $4k \times 4k$ binary matrix with at most $O(k^{4/3}/\log^{1/3}(k))$ 0-entries contains every $k \times k$ scattered permutation matrix.

We also further improve the upper bound on c_P to $c_P \leq 2^{(4+o(1))k}$, for all $k \times k$ permutation matrices P.

All the bounds mentioned here imply similar bounds on the Stanley-Wilf limit, s_P, since it is known that $c_P \leq O(s_P^{4.5})$ and $s_P \leq O(c_P^2)$ for every permutation matrix P.

On 2-Limited Packings of Complete Grid Graphs

Nancy E. Clarke, Acadia University, Wolfville Canada

For a fixed integer k, a set of vertices B of a graph G is a *k-limited packing* of G provided that the closed neighbourhood of any vertex in G contains at most k elements of B. The size of a largest possible k-limited packing in G is denoted $L_k(G)$ and is the k-limited packing number of G. In this paper, we investigate the 2-limited packing number of Cartesian products of paths. We show that the function $\Delta[L_2(P_k \Box P_n)] = L_2(P_k \Box P_n) - L_2(P_k \Box P_{n-1})$ is eventually periodic, and thereby give closed formulas for $L_2(P_k \Box P_n)$, $k = 1, 2, \ldots, 5$. The techniques we use are suitable for establishing other types of packing and domination numbers for Cartesian products of paths and, more generally, for graphs of the form $H \Box P_n$. This is joint work with R.P. Gallant.

Matroid union,
Graphic? Binary? Neither?

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Joint work with András Recski

Graphic matroids form one of the most significant classes in matroid theory. When introducing matroids, Whitney concentrated on relations to graphs. The definition of some basic operations like deletion, contraction and direct sum were straightforward generalizations of the respective concepts in graph theory. Most matroid classes, for example those of binary, regular or graphic matroids, are closed with respect to these operations. This is not the case for the union (also referred to as sum). The union of two graphic matroids can be nongraphic.

The first paper in this area was that of Lovász and Recski: they examined the case if several copies of the same graphic matroid are given. Then Recski conjectured thirty years ago that if the union of graphic matroids is not graphic then it is nonbinary. He also studied the case if we fix one simple graphic matroid and take its union with every possible graphic matroid.

If there are two matroids and the first one can be drawn as a graph with two points, then a necessary and sufficient condition is given for the other matroid to ensure the graphicity of the union. A similar case has been proved where the first matroid is a circuit with loops and bridges.

Theorem. *If $M(G_0)$ consists of loops and a single circuit of length n or n parallel edges ($n \geq 2$) and $M(G_1)$ is an arbitrary graphic matroid in the same ground set then the graphicity of the union can be decided in polynomial time.*

Applying some steps of the proof of this theorem we also prove that the above conjecture holds for these cases.

One can ask further questions about the classes formed by those graphic (or arbitrary) matroids whose union with any graphic (or arbitrary) matroid is graphic (or either graphic or nonbinary). These 2^3 variations define 8 matroid classes. We present some results about their relations and properties.

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Vizing's Theorem and Kőnig's Line Coloring Theorem for graphings

Endre Csóka, University of Warwick

(joint work with Gábor Lippner and Oleg Pikhurko)

Vizing's Theorem states that if the maximum degree of a graph is d, then its edge-chromatic number is at most $d+1$. Kőnig's Line Coloring Theorem states that for bipartite graphs, the edge-chromatic number is always d. We investigate the analogous questions for measurable graphs called graphings. We show that $d+O(\sqrt{d})$ is an upper bound for graphings, and $d+1$ is the sharp upper bound for bipartite graphings. We show that a generalization of Vizing's Theorem (for finite graphs) would imply that $d+1$ is an upper bound for non-bipartite graphings, as well.

Covering 2-edge-colored graphs with a pair of cycles

Louis DeBiasio, Miami University

(joint work with Luke Nelsen)

Lehel conjectured that in every 2-coloring of the edges of K_n, there is a vertex disjoint red and blue cycle which span $V(K_n)$. Łuczak, Rödl, and Szemerédi proved Lehel's conjecture for large n, Allen gave a different proof for large n, and finally Bessy and Thomassé gave a proof for all n. Balogh, Barát, Gerbner, Gyárfás, and Sárközy proposed a strengthening of Lehel's conjecture where K_n is replaced by any graph G with $\delta(G) \geq 3n/4$, and they proved an approximate version of their conjecture. We prove that their conjecture holds for sufficiently large n..

Maximum measures of spherical sets avoiding orthogonal pairs of points

Evan DeCorte, University of Delft

(joint work with Oleg Pikhurko.)

Let a_n be the supremum of the Lebesgue (surface) measure of I, where I ranges over all measurable sets of unit vectors in R^n such that no two vectors in I are orthogonal, and where the surface measure is normalized so that the whole sphere gets measure 1. The problem of determining a_n was first stated in a 1974 note by H. S. Witsenhausen, where he gave the upper bound of $1/n$ using a simple averaging argument. In a 1981 paper by Frankl and Wilson, they prove their well-known theorem and use it to attack this problem; there it was shown that a_n decreases exponentially. In this talk, we focus on the case $n = 3$, where we improve Witsenhausen's $1/3$ upper bound to 0.313. The proof involves some basic harmonic analysis and infinite-dimensional linear programming.

Rainbow copies of C_4 in edge-colored hypercubes

Michelle Delcourt, University of Illinois, Urbana-Champaign

(joint work with József Balogh, Bernard Lidick, and Cory Palmer.)

For positive integers k and d such that $4 \leq k < d$ and $k \neq 5$, we determine the maximum number of rainbow colored copies of C_4 in a k-edge-coloring of the d-dimensional hypercube Q_d. Interestingly, the k-edge-colorings of Q_d yielding the maximum number of rainbow copies of C_4 also have the property that every copy of C_4 which is not rainbow is monochromatic.

On Induced Paths, Holes and Trees in Random Graphs

Kunal Dutta, Max-Planck-Institut fr Informatik, Saarbrücken

(joint work with C. R. Subramanian)

We study the concentration of the largest induced paths, trees and cycles (holes) in the Erdos-Renyi random graph model and prove a 2-point concentration for the size of the largest induced path and hole, for all $p = \Omega(n^{?1/2} \ln^2 n)$. As a corollary, we obtain an improvement over a result of Erdos and Palka concerning the size of the largest induced tree in a random graph. Further, we study the path chromatic number and tree chromatic number i.e. the smallest number of parts into which the vertex set of a graph can be partitioned such that every The arguments involve the application of a modified version of a probabilistic inequality of Krivelevich, Sudakov, Vu and Wormald.

Linear Forests on Hamiltonian Cycles

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Abstract

Given integers k, s, t with $0 \leq s \leq t$ and $k \geq 0$, a (k, t, s)-linear forest F is a graph that is the vertex disjoint union of t paths with a total of k edges and with s of the paths being single vertices. Given integers m and n with $k + t \leq m \leq n$, a graph G of order n is (k, t, s, m)-pancyclic if for any (k, t, s)-linear forest F and for each integer r with $m \leq r \leq n$, there is a cycle of length r containing the linear forest F. If the paths of the forest F are required to appear on the cycle in a specified order, then the graph is said to be (k, t, s, m)-pancyclic ordered. If, in addition, each path in the system is oriented and must be traversed in the order of the orientation, then the graph is said to be strongly (k, t, s, m)-pancyclic ordered. Minimum degree conditions and minimum sum of degree conditions of nonadjacent vertices that imply a graph is (k, t, s, m)-pancylic, as well as degree conditions that imply a graph is (strongly) (k, t, s, m)-pancylic ordered will be given. Examples showing the sharpness of the conditions will be described. Also, minimum degree conditions that imply fixed vertices can be placed on Hamiltonian cycle at predetermined distances will be presented. Problems and open questions related to these conditions will be presented.

Extremal Combinatorics, Geometry, and Algebra

Jacob Fox, Massachusets Institute of Technology

Famous Ramsey, Turán, and Szemerédi-type results prove the existence of certain patterns in graphs and hypergraphs under mild assumptions. We survey recent results of János Pach and his collaborators which have shown much stronger results for graphs and hypergraphs that arise from geometry or algebra.

$\mathbb{Z}_2$-embeddings of Clustered Graphs

Rado Fulek

(joint work with J. Kynčl, I. Malinović and D. Pálvölgyi)

Hanani–Tutte theorem is a classical result proved for the first time in 1930s that characterizes planar graphs as graphs that admit a drawing in the plane in which every pair of edges not sharing a vertex cross an even number of times. We generalize Hanani–Tutte theorem to clustered graphs with two disjoint clusters, and show that a straightforward extension of our result to flat clustered graphs with three or more disjoint clusters is not possible. Similarly as Hanani-Tutte theorem, our generalization gives a polynomial-time algorithm for clustered planarity testing in the case of two clusters. We also discuss possible extensions of our results and their consequences for other variants of planarity.

Extremal results for Berge-hypergraphs

Dániel Gerbner, MTA Rényi Institute

(joint work with Cory Palmer)

Let H be a hypergraph and G be a graph. We say that H contains G if we can embed G into the vertex set of H such that each edge of G can be associated with a distinct edge of H containing it. We say H is G-free if it does not contain G. (When H is a graph this is the ordinary notion that H does not contain G as a subgraph).

We would like to determine the maximum possible size of the sum of the vertex degrees in an G-free hypergraph H on n vertices. (When H is a graph this maximum is twice the extremal number of G). Győri and Lemons showed that for 3-uniform hypergraphs, when G is an even cycle that this maximum has the same order as the extremal number of even cycle in graphs. Surprisingly, for cycles of length $2k+1$ the parameter is the same order as for cycles of length $2k$ (this is significantly different from the extremal number of odd cycles in graphs). We examine this question in a slightly more general setting and show that for any graph G, the maximum degree sum cannot behave too differently from the extremal number of G. We then focus on the particular case when G is a complete bipartite graph to get an analogue of the Kővari-Sós-Turán theorem.

Maximum density of exact copies of a graph in the n-cube and a Turán surprise.

John Goldwasser, West Virginia University

Let G be an induced subgraph of the d-cube Q_d. We define $f(d, G)$, the d-cube density of G, to be the limit as n goes to infinity of the maximum fraction, over all subsets J of the vertex set of the n-cube Q_n, of sub-d-cubes of Q_n whose intersection with J induces an exact copy of G (isomorphic to G, with the same embedding in Q_d). In general, it is difficult to determine $f(d, G)$. We show that if C is a perfect 8-cycle (4 pairs of vertices at distance 4) then $f(4, C) = 3/32$. Amazingly, to establish the upper bound we needed to determine the Turán density of $\{F, H\}$, where $F = \{1234, 1235, 1245\}$ and $G = \{1234, 1235, 1456\}$ and where the only 4-graphs allowed are those where there is a bipartition of the vertex set such that each edge has two vertices in each part. We note that the link graphs of the vertex 1 in F and G are the two forbidden 3-graphs in Bollobas well-known theorem on the maximum number of edges in a 3-graph where no edge contains the symmetric difference of two others.

ANTI-RAMSEY NUMBERS IN COMPLETE SPLIT GRAPHS

IZOLDA GORGOL

A subgraph of an edge-coloured graph is *rainbow* if all of its edges have different colours. For graphs G and H the *anti-Ramsey number* $ar(G, H)$ is the maximum number of colours in an edge-colouring of G with no rainbow copy of H. The notion was introduced by Erdős, Simonovits and V. Sós and studied in case $G = K_n$. Afterwards exact values or bounds for anti-Ramsey numbers $ar(K_n, H)$ were established for various H among others by Alon, Jiang & West, Montellano-Ballesteros & Neumann-Lara, Schiermeyer. There are also results concernig bipartite graphs, cubes or product of cycles as G obtained by Axenovich, Li, Montellano-Ballesteros, Schiermeyer and others. In the talk we give the survey of these results and also there will be presented numerous results with a complete split graph $K_n + \overline{K}_m$ as the host graph G.

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Key words and phrases. anti-Ramsey number, rainbow number.

The clique, independence and chromatic numbers of random subgraphs of distance graphs

A.S. Gusev, M.M. Pyaderkin, A.M. Raigorodskii

Our talk is concerned with the classical Nelson–Hadwiger problem on finding the chromatic numbers of distance graphs in $\mathbb{R}^n$. We introduce a class of graphs $G(n,r,s) = (V(n,r), E(n,r,s))$ defined as follows:

$$V(n,r) = \{x = (x_1, x_2, \ldots, x_n) : \ x_i \in \{0,1\}, \ x_1 + x_2 + \ldots + x_n = r\}, \quad E(n,r,s) = \{\{x,y\} : (x,y) = s\},$$

where (x,y) is the Euclidean scalar product.

We study the random graphs $\mathcal{G}(G(n,r,s),p)$ whose edges are chosen independently from the set $E(n,r,s)$ each with probability p. We obtain sharp asymptotic bounds for the clique, independence and chromatic numbers of such graphs depending on some relations between the parameters n, r, s.

Score and imbalance sets of multigraphs

Antal Iványi, Zoltán Kása, Shariefuddin Pirzada

We consider two problems:

1. How to test the potential score sets of multigraphs, how many score sets have the multigraphs, and how to reconstruct the corresponding graphs?
2. How to test the potential imbalance sets of directed multigraphs, how many imbalance sets have the directed multigraphs and how to reconstruct the corresponding graphs?

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A hypergraph Turán theorem via a generalised notion of hypergraph Lagrangian.

Matthew Jenssen, London School of Economics

The theory of hypergraph Lagrangians, developed by Frankl and Füredi [Bull. Inst. Math. Acad. Sin. **16** (1988), 305–313] and Sidorenko [Mat. Zametki **41** (1987), 433–455], is a valuable tool in the field of hypergraph Turán problems. Here we present a generalised notion of the hypergraph Lagrangian and use the Karush-Kuhn-Tucker conditions from the theory of non-linear programming to exploit some of it's properties. As an application we show that the maximum Lagrangian of an r-graph H with the property that for all $e, f \in E(H), e \cap f \neq r-2$ is attained by $K_{r+1}^{(r)}$, the complete r-graph on $r+1$ vertices in the cases $r = 3, 4, 5, 6, 7$and 8. As a consequence we determine the Turán density of what we shall call the 'r-uniform generalised K_4' for these values of r. More precisely, the r-uniform generalised K_4, denoted by $\mathcal{K}_4^{(r)}$, is the r-graph on the $5r-6$ vertices $\{x_i, y_j, z_{ijk} : i = 1, \ldots, r, j = 1, 2, k = 1, \ldots r-2\}$ and with the 6 edges

$$\{x_1, \ldots, x_r\}, \{y_1, y_2, x_3, \ldots, x_r\} \text{ and } \{x_i, y_j, z_{ij1}, \ldots, z_{ij(r-2)}\} \text{ for } i, j \in \{1, 2\}.$$

We note that $\mathcal{K}_4^{(2)} = K_4$, the complete graph on 4 vertices, so that the above results may be viewed as hypergraph extensions of known Turán results on K_4. The generalised K_4 is naturally related to the generalised triangle, whose Turán density is considered (either implicitly or explicitly) in the works of Frankl and Füredi [J. Combin. Theory Ser. A **52** (1989), 129–147] and Pikhurko [Combinatorica **28** (2008) 187–208] amongst others.

Convex Polygons are Self-Coverable

Balázs Keszegh, MTA Rényi Institute of Mathematics
(joint work with Dömötör Pálvölgyi)

We introduce a new notion for geometric families called self-coverability and show that homothets of convex polygons are self-coverable. As a corollary, we obtain several results about coloring point sets such that any member of the family with many points contains all colors. This is dual (and in some cases equivalent) to the much investigated cover-decomposability problem.

Polynomial Time Algorithms for the 3-Dimensional VLSI Routing in the Cube

Attila Kiss[1,2], András Recski[1,3]

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Abstract. In previous works some polynomial time algorithms were presented for special cases of the 3-Dimensional VLSI Routing problem. Solutions were given to problems when all the terminals are either on a single face (SALP - Single Active Layer Problem) or on two opposite faces (3DCRP - 3-Dimensional Channel Routing Problem) or on two adjacent faces (3DΓRP - 3-Dimensional Gamma Routing Problem) of a rectangular cuboid. We prove that combining these algorithms one can solve any given problem on cubes and we give some polynomial time algorithms to find these solutions.

Keywords: VLSI design, 3-dimensional routing, Steiner-tree

Routing in the design of VLSI (Very Large Scale Integrated) circuits is an important area of modern applied mathematics, in particular combinatorial optimization. There were a lot of interesting results in this area in the last four decades. Although more and more problems are proved to be NP-complete, there are a lot of heuristic solutions, approximating the optimum of these problems with a good rate of efficiency (for a further view read [1]). There are many well known technologies to construct electric circuits for this model. We give a new theoretical model that can be a future direction in the development of new circuits. We would like to construct "routing boxes" that means one can place terminals in all the faces of a rectangular cuboid formed by the circuit boards layered together.
From a graph-theoretical point of view the 2-dimensional detailed routing problems (in particular, the most often studied channel routing and switchbox routing problems) search for vertex-disjoint Steiner-trees (trees with given sets containing specific terminals) on a (2-dimensional) square grid while the 3-dimensional ones search on a (3-dimensional) cu-

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bic grid.
Since even the Channel Routing Problem (when all the terminals are on two opposite boundaries of the square grid) cannot always be solved, one has to introduce several parallel layers. In the last four decades hundreds or perhaps thousands of papers studied the possibilities of routing a channel or switchbox using a possibly small number of layers. However, no universal constant exists for the number of layers to make every switchbox routing (with any size and shape) possible.
Similarly, even the simplest 3-dimensional problem (the Single Active Layer Routing Problem, when all the terminals are on a single external face of the cubic grid) cannot always be solved, one has to make the grid "finer". A *spacing* of size s in a given direction means that we introduce s extra lines between any two consecutive lines in that direction (plus s extra lines after the last original one). In a complete analogy with the 2-dimensional case, no universal refinement (that is, maximal spacing size) will do for 3-dimensional switchboxes of any size and shape.
In this paper we prove the existence of a universal spacing size if the switchbox is a cube (of arbitrary size). Our upper bound will be of theoretical interest only but we expect that it can drastically be reduced (we successfully decreased the size of the spacing in some special cases).
To our best knowledge three special cases of the 3-dimensional switchbox routing problem have already been studied, namely: Single Active Layer Routing Problem [2],[3], 3-Dimensional Channel Routing Problem [4], 3-Dimensional Γ Routing Problem [5],[6]. In addition to these three subproblems we have six further cases. We proved the following:

Theorem 1. *Combining the methods used by the three previous subproblems all the six new cases on a cube can be solved with a fixed maximum number of spacings needed. Our solution can be found in polynomial time.*

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POLYNOMIAL-TIME PERFECT MATCHINGS IN DENSE HYPERGRAPHS

PETER KEEVASH, FIACHRA KNOX AND RICHARD MYCROFT

In this talk we consider the decision problem for the existence of a perfect matching in a k-uniform hypergraph (or k-graph) H on n vertices. Since for $k \geq 3$ this problem was one of Karp's 21 NP-complete problems [1], it is natural to seek conditions on H which render it tractable. For any $A \subseteq V(H)$, the *degree* $d(A) = d_H(A)$ of A is the number of edges of H containing A. The *minimum* $(k-1)$*-degree* $\delta_{k-1}(H)$ of H is the minimum of $d(A)$ over all subsets A of $V(H)$ of size $k-1$.

Let $\mathbf{PM}(k, \delta)$ be the decision problem of determining whether a k-graph H with $\delta_{k-1}(H) \geq \delta n$ contains a perfect matching. Szymańska [3] proved that for $\delta < 1/k$ the problem $\mathbf{PM}(k, 0)$ admits a polynomial-time reduction to $\mathbf{PM}(k, \delta)$ and hence $\mathbf{PM}(k, \delta)$ is also NP-complete. We describe an algorithm which shows that the opposite is true for $\delta > 1/k$:

Theorem 1. *Fix $k \geq 3$ and $\gamma > 0$. Then there is an algorithm with running time $O(n^{3k^2-7k+1})$, which given any k-graph H on n vertices with $\delta_{k-1}(H) \geq (1/k + \gamma)n$, finds either a perfect matching or a certificate that no perfect matching exists.*

Previously, Karpiński, Ruciński and Szymańska [2] showed that there exists $\varepsilon > 0$ such that $\mathbf{PM}(k, 1/2 - \varepsilon)$ is in P.

To prove Theorem 1 we establish a strong stability result which states that if H is a k-graph on n vertices, and $\delta_{k-1}(H) \geq n/k + o(n)$, then H either contains a perfect matching or is close to one of a family of lattice-based constructions termed 'divisibility barriers'. While the precise statement of this result for general k requires significant preliminaries, which we cover in the talk, the special case $k = 3$ may be stated as follows:

Theorem 2. *For any $\gamma > 0$ there exists $n_0 = n_0(\gamma)$ such that the following statement holds. Let H be a 3-graph on $n \geq n_0$ vertices, such that 3 divides n and $\delta_2(H) \geq (1/3 + \gamma)n$, and suppose that H does not contain a perfect matching. Then there is a subset $A \subseteq V(H)$ such that $|A|$ is odd but every edge of H intersects A in an even number of vertices.*

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$K_{(s,t)}$-saturated bipartite graphs

Dániel Korándi, ETH Zürich

(joint work with Wenying Gan and Benny Sudakov)

An n-by-n bipartite graph is H-saturated if the addition of any missing edge between its two parts creates a new copy of H. In 1964, Erdős, Hajnal and Moon made a conjecture on the minimum number of edges in a $K_{(s,s)}$-saturated bipartite graph. This conjecture was proved independently by Wessel and Bollobás in a more general, but ordered, setting: they showed that the minimum number of edges in a $K_{(s,t)}$-saturated bipartite graph is $n^2-(n-s+1)(n-t+1)$, where $K_{(s,t)}$ is the "ordered" complete bipartite graph with s vertices in the first color class and t vertices in the second. However, the very natural question of determining the minimum number of edges in the unordered $K_{(s,t)}$-saturated case remained unsolved. This problem was considered recently by Moshkovitz and Shapira who also conjectured what its answer should be. We give a bound on the minimum number of edges in a $K_{(s,t)}$-saturated bipartite graph that is only smaller by an additive constant than the conjectured value. In this talk we sketch the ideas behind the proof.

Turán problems and shadows

Alexandr Kostochka, University of Illinois Urbana-Champaign
(joint work with Dhruv Mubayi and Jacques Verstraëte)

The *Turán number*, $\mathrm{ex}_r(n, F)$ of an r-uniform hypergraph F is the maximum number of edges in an r-uniform hypergraph with n vertices not containing copies of F. All the four Honorees of the meeting have strong results on Turán numbers. The expansion G^+ of a graph G is the 3-uniform hypergraph obtained from G by enlarging each edge of G with a vertex disjoint from $V(G)$ such that distinct edges are enlarged by distinct vertices.
We determine $\mathrm{ex}_3(n, G^+)$ exactly when G is a path or cycle, thus settling conjectures of Füredi and Jiang (for cycles) and Füredi, Jiang and Seiver (for paths). We find the asymptotics for $\mathrm{ex}_3(n, G^+)$ when G is any fixed forest. This settles a conjecture of Füredi. We also show that for each graph G, either $\mathrm{ex}_3(n, G^+) \leq \left(\frac{1}{2} + o(1)\right) n^2$ or $\mathrm{ex}_3(n, G^+) \geq (1 + o(1))n^2$, thereby exhibiting a jump for the Turán number of expansions. In addition, for the graph Q_3 of the 3-dimensional unit cube, we show $\mathrm{ex}_3(n, Q_3) = \Theta(n^2)$.

Title: Randomness in Maker-Breaker games

Gal Kronenberg
Tel Aviv University

Abstract: We consider two random versions of Maker-Breaker games.
The first setting is the **random-turn** Maker-Breaker games, firstly introduced by Peres, Schramm, Sheffield and Wilson in 2007. A *p-random-turn Maker-Breaker game* is the same as an ordinary Maker-Breaker game, except that instead of alternating turns, the players toss a coin before each turn to decide the identity of the next player to move (the probability of Maker to move is p).
In the second setting we consider the biased **random-player** Maker-Breaker game. In this version, one of the players plays according to an optimal strategy, while the other plays randomly. Under this setting we actually have two different games: the $(m : 1)$ random-Maker game and the $(1 : b)$ random-Breaker game. We call m and b the bias of the game.
We analyze the two random versions of several classical games such as the game Box (introduced by Chvátal and Erdős in 1987), the Hamilton cycle game and the k-connectivity game (both played on the edge set of K_n). For each such game, we show an efficient strategies for the typical winner of the game.

Joint work with: Asaf Ferber and Michael Krivelevich.

Spanning quadrangulations of triangulated surfaces

André Kündgen,
(joint work with Carsten Thomassen)

While on any fixed surface there are only finitely many minimal graphs that are not 5-vertex-colorable, there is no such characterization for 4-vertex-coloring on any surface other than the sphere. On the positive side, a triangulation of a surface is 4-vertex-colorable if and only if the edges can be labeled with 3 colors such that the union of any two color classes forms a bipartite spanning quadrangulation. We explore this idea by establishing connections between spanning quadrangulations and cycles in the dual graph which are noncontractible and alternating with respect to a perfect matching.
We show that the dual graph of an Eulerian triangulation of an orientable surface other than the sphere has a perfect matching M and an M-alternating noncontractible cycle. As a consequence, every Eulerian triangulation of the torus has a nonbipartite spanning quadrangulation. For an Eulerian triangulation G of the projective plane the situation is different: If the dual graph of G is nonbipartite, then it has no noncontractible alternating cycle, and all spanning quadrangulations of G are bipartite. If the dual graph of G is bipartite, then it has a noncontractible, M-alternating cycle for some (and hence any) perfect matching M, and thus G has a bipartite spanning quadrangulation and also a nonbipartite spanning quadrangulation.

On random subgraphs of a Kneser graph.

Andrey Kupavskii*

June 17, 2014

Our talk is devoted to the study of a Kneser graph $KG_{n,k}$. The vertices of the graph are the k-subsets of n-element set. Two k-sets are joined by an edge if they are disjoint. These graphs were first investigated by Martin Kneser [3]. He showed that $\chi(KG_{n,k}) \le n - 2k + 2$ and conjectured that this bound is tight. The conjecture was proved by László Lovász [4] over 20 years later. He used tools from algebraic topology, giving birth to the field of topological combinatorics. Later, a very nice and short proof was given by Joshua E. Greene [2].

Several papers we devoted to the study of the chromatic number of Kneser graphs of set systems. For any system of k-sets $\mathcal{A} \subset \binom{[n]}{k}$ we can define the Kneser graph $KG(\mathcal{A})$ in the following natural way. The vertices of $KG(\mathcal{S})$ are the elements of $\mathcal{A}$, while two of them are joined if and only if they are disjoint. In particular, there were results by Dolnikov [1] and Schrijver [5].

Such graphs are induced subgraphs of $KG_{n,k}$. We, in turn, study spanning subgraphs of $KG_{n,k}$. Namely, we study the chromatic number of a random graph $KG_{n,k}(p)$. This graph has the same set of vertices as $KG_{n,k}$, and each edge from $KG_{n,k}$ is included in $KG_{n,k}(p)$ with probability p. Informally, for a large range of values of parameters we show that the chromatic number of the random subgraph is w.h.p. close to the chromatic number of the original graph. In particular, we have the following

Theorem. *1. If p is fixed, $0 < p \le 1$, and $k \gg n^{\frac{3}{2l}}$, then w.h.p. $\chi(KG_{n,k}(p)) \ge \chi(KG_{n,k}) - 2l$.*
2. If for some $p = p(n)$, $0 < p \le 1$, we have $k \gg n^{3/4}p^{-1/4} + (n^{1/2}\ln n)p^{-1/2}$, then w.h.p. $\chi(KG_{n,k}(p)) \ge \chi(KG_{n,k}) - 4$.
3. Let p is fixed, $0 < p \le 1$, and $n - 2k = \bar{o}(\sqrt{n})$, then w.h.p. $\chi(KG_{n,k}(p)) \ge \chi(KG_{n,k}) - 2$.

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Graphs with and without cycles

Nathan Lemons, Los Alamos National Laboratory

A well known question of Erdős asks: How many pentagons can there be in a triangle free graph? We consider a couple of variants to this problem in both graphs and hypergraphs. We discuss a connection to the problem of relating the extremal sizes of graphs excluding a cycle of length $2k$ and graphs of girth at least $2k+1$.

Intersecting k-uniform families containing all the k-subsets of a given set

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Abstract

Let m, n, and k be integers satisfying $0 < k \leqslant n < 2k \leqslant m$. A family of sets $\mathcal{F}$ is called an (m, n, k)-*intersecting family* if $\binom{[n]}{k} \subseteq \mathcal{F} \subseteq \binom{[m]}{k}$ and any pair of members of $\mathcal{F}$ have nonempty intersection. Maximum (m, k, k)- and $(m, k+1, k)$-intersecting families are determined by the theorems of Erdős-Ko-Rado and Hilton-Milner, respectively. We determine the maximum families for the cases $n = 2k-1, 2k-2, 2k-3$, or m sufficiently large.

Joint work with Bor-Liang Chen, Kuo-Ching Huang, and Ko-Wei Lih.

On the number of K_4-saturating edges

Hong Liu, UIUC

(Joint work with József Balogh.)

Let G be a K_4-free graph, an edge in its complement is a K_4-*saturating* edge if the addition of this edge to G creates a copy of K_4. Erdős and Tuza conjectured that for any n-vertex K_4-free graph G with $\lfloor n^2/4 \rfloor + 1$ edges, one can find at least $(1+o(1))\frac{n^2}{16}$ K_4-saturating edges. We construct a graph with only $\frac{2n^2}{33}$ K_4-saturating edges. Furthermore, we prove that it is best possible, i.e., one can always find at least $(1+o(1))\frac{2n^2}{33}$ K_4-saturating edges in an n-vertex K_4-free graph with $\lfloor n^2/4 \rfloor + 1$ edges.

New bounds for 3-part Sperner-families

András Mészáros, ELTE

We give a new upper and lower bound on the size of maximum 3-part Sperner families. We prove that $1,05 < d_3 < 1,072$. Further we disprove a conjecture of Aydinian, Czabarka, Erdős, Székely on the maximum size of k-part Sperner families for the case of equal parts of size $2^\ell - 1$.

Cyclic chain decomposition method for forbidding subposets

Abhishek Methuku, CEU

(joint work with Casey Tompkins.)

We introduce a new method for determining the size of the largest family of subsets of $[n]$ not containing any of one or more given posets as a weak subposet. The method involves decomposing the set of intervals along a cyclic permutation into chains of a certain type called as "Oscillating chains" and considering the interaction between consecutive Oscillating chains. In particular, we consider a poset which strictly contains the butterfly poset as a subposet and show that the same bound holds, thereby generalizing a result of DeBonis, Katona and Swanepoel. We determine $La(n, P_1, P_2)$ for an infinite set of pairs (P_1, P_2), one of which provides a second generalization of their result. Other possible decompositions of the set of intervals and a conjecture will be presented.

On Saturated k-Sperner Systems

Natasha Morrison, Oxford University

(joint work with Jonathan Noel and Alex Scott)

Given a set X, a collection $\mathcal{F} \subseteq \mathcal{P}(X)$ is said to be *k-Sperner* if it does not contain a chain of length $k+1$ under set inclusion and it is *saturated* if it is maximal with respect to this property. Gerbner, Keszegh, Lemons, Palmer, Pálvölgyi and Patkós conjectured that, if $|X|$ is sufficiently large with respect to k, then the minimum size of a saturated k-Sperner system $\mathcal{F} \subseteq \mathcal{P}(X)$ is 2^{k-1}. In this talk we disprove this conjecture by showing that there exists $\varepsilon > 0$ such that for every k and $|X| \geq n_0(k)$ there exists a saturated k-Sperner system $\mathcal{F} \subseteq \mathcal{P}(X)$ with cardinality at most $2^{(1-\varepsilon)k}$.

Perfect packings in hypergraphs

Richard Mycroft, University of Birmingham

Let G and H be graphs or k-graphs (k-uniform hypergraphs). Then a perfect H-packing in G is a collection of vertex-disjoint copies of H in G which together cover all vertices of G. For graphs, the minimum degree condition needed to ensure the existence of a perfect H-packing in G was considered by several authors, before finally Kühn and Osthus gave a condition for any graph H which is best-possible up to an additive constant. However, very few analogous results for k-graphs are known outside the case of a perfect matching (when H consists of a single edge). In this talk I will outline some recent developments for this problem.

Union-intersecting set systems

Dániel T. Nagy
Eötvös Loránd University, Budapest

Joint work with Gyula O.H. Katona

Three intersection theorems are proved. First, we determine the size of the largest set system, where the system of the pairwise unions is l-intersecting. Then we investigate set systems where the union of any s sets intersect the union of any t sets. The maximal size of such a set system is determined exactly if $s + t \leq 4$, and asymptotically if $s + t \geq 5$. Finally, we exactly determine the maximal size of a k-uniform set system that has the above described (s, t)-union-intersecting property, for large enough n.

Saturation Games

Alon Naor, Tel Aviv University

Let $\mathcal{P}$ be a monotone increasing graph property and let G be a graph on n vertices which does not satisfy $\mathcal{P}$. An edge $e \in K_n \setminus G$ is called *legal* (with respect to G and $\mathcal{P}$) if $G \cup \{e\}$ does not satisfy $\mathcal{P}$. In the saturation game $(n, \mathcal{P})$ two players, called Shorty and Prolonger, build together a graph $G \subseteq K_n$ which does not satisfy $\mathcal{P}$. Shorty and Prolonger take turns is claiming legal edges (starting from the empty graph on n vertices) until none exist. At this point the game is over, and the resulting graph G is said to be $\mathcal{P}$ *saturated*. The *score* of the game is the number of edges in G at the end of the game. Shorty's goal is to minimize the score of the game, while Prolonger's goal it to maximize the score of the game.

We analyze saturation games for several graph properties, including $\mathcal{P} =$ "having chromatic number at least k" and $\mathcal{P} =$ "containing a k-matching", showing some surprising results.

Joint work with Dan Hefetz, Michael Krivelevich and Miloš Stojaković.

Exactly m-coloured graphs

Bhargav Narayanan, Cambridge University

(joint work with Teeradej Kittipassorn)

Given an edge-colouring of a graph with a set of m colours, we say that the graph is exactly m-coloured if each of the colours is used. If we are given an edge-colouring of the complete graph on the natural numbers with infinitely many colours, for which numbers m can one always find an exactly m-coloured complete subgraph? Stacey and Weidl asked this question in 1999, noting that the injective colouring leaves only numbers of the form $n(n-1)/2$ as potential candidates. Teeradej Kittipassorn and I answered this question recently; we proved that whenever the complete graph on the natural numbers is coloured with infinitely many colours, there is a complete $(n(n-1)/2)$-coloured subgraph for every natural number n. In this talk, I will talk about this theorem and various other related questions and results.

Some Covering Problems in Geometry

Márton Naszódi, ELTE

We discuss variations of the following problem: given a set in Euclidean n-space (resp. on the sphere). Bound the minimum number of translates (resp. rotated copies) that cover another given set (resp. the sphere). We present a method to obtain upper bounds for these problems. As applications of this method, we generalize some results of Rogers, and sharpen an estimate by Artstein–Avidan and Slomka. The key idea which makes our proofs rather simple and uniform throughout distinct geometric settings is the application of an algorithmic result of Lovász as opposed to the probabilistic approach taken by others.
If time permits, we discuss a lower bound, too. We consider the illumination problem (the problem of covering a convex body by translates of its interior). By a probabilistic argument, we show that arbitrarily close to the Euclidean ball there is a centrally symmetric convex body of illumination number exponentially large in the dimension.

List Colourings of Graphs on a Bounded Number of Vertices

Jonathan Noel, Oxford University

(joint work with Bruce Reed, Doug West, Hehui Wu and Xuding Zhu.)

The choice number (also called list chromatic number) of a graph G is the minimum integer k such that for any assignments of lists of size k to the vertices of G, there is a proper colouring of G in which every vertex is mapped to a colour in its list. For general graphs, the choice number is not bounded above by any function of the chromatic number.

In this talk, we will discuss a proof of Ohba's Conjecture, which states that if the number of vertices in G is bounded above by $2\chi(G)+1$, then the choice number of G is equal to its chromatic number. Moreover, we will provide a generalisation of this result which gives a tight upper bound on the choice number of graphs with at most 3χ vertices. We will conclude the talk by posing several open problems for future study.

Recency-based preferential attachment models

Liudmila Ostroumova[1,2], Egor Samosvat[1,3]

Preferential attachment models were shown to be very effective in predicting such important properties of real-world networks as the power-law degree distribution, small diameter, etc. Many different models are based on the idea of preferential attachment: LCD, Buckley-Osthus, Holme-Kim, fitness, random Apollonian network, and many others.

Although preferential attachment models reflect some important properties of real-world networks, they do not allow to model the so-called *recency property*. Recency property reflects the fact that in many real networks nodes tend to connect to other nodes of similar age. This fact motivated us to introduce a new class of models – recency-based models. This class is a generalization of fitness models, which were suggested by Bianconi and Barabási. Bianconi and Barabási extended preferential attachment models with pages' inherent quality or *fitness* of nodes. When a new node is added to the graph, it is joined to some already existing nodes that are chosen with probabilities proportional to the product of their fitness and incoming degree.

We generalize fitness models by adding a recency factor to the attractiveness function. This means that pages are gaining incoming links according to their *attractiveness*, which is determined by the incoming degree of the page, its *inherent popularity* (some page-specific constant) and age (new pages are gaining new links more rapidly).

We analyze different properties of recency-based models. For example, we show that some distributions of inherent popularity lead to the power-law degree distribution.

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Generalized multiplicative Sidon-sequences

Péter Pál Pach
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As a generalization of multiplicative Sidon-sequences we investigate the following question: What is the maximal number of elements which can be chosen from the set $\{1, 2, \ldots, n\}$ in such a way that the equation $a_1 a_2 \ldots a_k = b_1 b_2 \ldots b_k$ does not have a solution of distinct elements? Let us denote this maximal number by $G_k(n)$. Erdős studied the case $k = 2$: In 1938 he proved that $\pi(n) + c_1 n^{3/4}/(\log n)^{3/2} \leq G_2(n) \leq \pi(n) + c_2 n^{3/4}$ and 31 years later improved the upper bound to $\pi(n) + c_2 n^{3/4}/(\log n)^{3/2}$. Hence, in the lower- and upper bounds for $G_2(n)$ not only the main terms are the same, but the error terms only differ by a constant factor. We study $G_k(n)$ for $k > 2$, give asymptotically precise bounds for every k, and prove some estimates on the error terms.

To estimate $G_k(n)$ extremal graph theoretic results are used, namely results about the maximal number of edges of C_{2k}-free graphs and of such C_{2k}-free bipartite graphs, where the number of vertices in the two classes are fixed.

Note that our question is strongly connected to the following problem: Erdős, Sárközy, T. Sós and Győri investigated how many numbers can be chosen from $\{1, 2, \ldots, n\}$ in such a way that the product of any $2k$ of them is not a perfect square. The maximal size of such a subset is denoted by $F_{2k}(n)$. The functions F and G clearly satisfy the inequality $F_{2k}(n) \leq G_k(n)$.

Product irregularity strength of graphs and hypergraphs

Cory Palmer, University of Montana

(joint work with Jaehoon Kim)

Let G be a graph with no isolated edges and consider an edge-labeling $w : E(G) \to \{1, 2, 3, \ldots, s\}$ of the edges of G. The product degree $pd(v)$ of a vertex v is the product of weights on edges incident to v, i.e., $pd(v) = \prod_{e \ni v} w(e)$. We call an edge-labeling *product-irregular* if all *product degrees* in G are distinct. The minimal s such that there exists a product-irregular labeling of G with labels $\{1, 2, \ldots, s\}$ is the *product irregularity strength* of G. This parameter was introduced by Anholcer who determined the value for some specific classes of graphs and proved general upper and lower bounds. We establish improved upper and lower bounds on this parameter for graphs. In fact, our results hold for multi-hypergraphs (subject to some basic constraints).

Indecomposable coverings with unit discs

Dömötör Pálvölgyi, ELTE

We disprove the 1980 conjecture of János Pach about the cover-decomposability of open convex sets by showing that the unit disc is not cover-decomposable. In fact, our proof easily generalizes to any set with a smooth boundary. We also show that (the suitable variant of) the conjecture holds for unbounded sets.

Three-monotone interpolation*

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Abstract

A function $f\colon \mathbb{R} \to \mathbb{R}$ is called *k-monotone* if it is $(k-2)$-times differentiable and its $(k-2)$nd derivative is convex. A point set $P \subset \mathbb{R}^2$ is *k-monotone interpolable* if it lies on a graph of a k-monotone function. These notions have been studied in analysis, approximation theory etc. since the 1940s.

We show that 3-monotone interpolability is very non-local: we exhibit an arbitrarily large finite P for which every proper subset is 3-monotone interpolable but P itself is not. On the other hand, we prove a Ramsey-type result: for every n there exists N such that every N-point P with distinct x-coordinates contains an n-point Q such that Q or its vertical mirror reflection are 3-monotone interpolable. The analogs for k-monotone interpolability with $k = 1$ and $k = 2$ are classical theorems of Erdős and Szekeres, while the cases with $k \geq 4$ remains open.

We also investigate the computational complexity of deciding 3-monotone interpolability of a given point set. Using a known characterization, this decision problem can be stated as an instance of polynomial optimization and reformulated as a semidefinite program. We exhibit an example for which this semidefinite program has only doubly exponentially large feasible solutions, and thus known algorithms cannot solve it in polynomial time. While such phenomena have been well known for semidefinite programming in general, ours seems to be the first such example in polynomial optimization, and it involves only univariate quadratic polynomials.

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Choosability of Graph Powers

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Abstract

Recently, Kim and Park have found an infinite family of graphs whose squares are not chromatic-choosable. Xuding Zhu asked whether there is some k such that all kth power graphs are chromatic-choosable. We answer this question in the negative. We show that there is a positive constant c such that for any k there is a family of graphs G with $\chi(G^k)$ unbounded and $\chi_\ell(G^k) \geq c\chi(G^k) \log \chi(G^k)$.

Sharp Bounds on Davenport-Schinzel Sequences of Every Order

Seth Pettie

A Davenport-Schinzel sequence with order s is a sequence over an n-letter alphabet that avoids subsequences of the form $a..b..a..b..$ with length $s+2$. They were originally used to bound the complexity of the lower envelope of degree-s polynomials or any class of functions that cross at most s times. They have numerous applications in discrete geometry and the analysis of algorithms.
Let $DS_s(n)$ be the maximum length of such a sequence. In this talk I'll present a new method for obtaining sharp bounds on $DS_s(n)$ for every order s. This work reveals the unexpected fact that sequences with odd order s behave essentially like even order $s-1$. The results refute both common sense and a conjecture of Alon, Kaplan, Nivasch, Sharir, and Smorodinsky [2008]. Prior to this work, tight upper and lower bounds were only known for s up to 3 and even $s>3$.

On the union of arithmetic progressions

Rom Pinchasi, Technion

(joint work with Shoni Gilboa)

We show that for every $\epsilon > 0$ there is an absolute constant $c(\epsilon) > 0$ such that the following is true: The union of any n arithmetic progressions, each of length n, with pairwise distinct differences must consist of at least $c(\epsilon)n^{2-\epsilon}$ elements. We show also that this type of bound is essentially best possible, as we can find n arithmetic progressions, each of length n, with pairwise distinct differences such that the cardinality of their union is $o(n^2)$.
We develop some number theoretical tools that are of independent interest. In particular we give almost tight bounds on the following question: Given n distinct integers $a_1, ..., a_n$ at most how many pairs satisfy $a_j/a_i \in [n]$? More tight bounds on natural related problems will be presented.

1 Nonnegative k-sums in a set of numbers. Alexey Pokrovskiy

Suppose that we have a set of numbers $x_1, \ldots, x_n$ which have nonnegative sum. How many subsets of k numbers from $\{x_1, \ldots, x_n\}$ must have nonnegative sum?

By choosing $x_1 = n-1$ and $x_2 = \cdots = x_n = -1$ we see that the answer to this question can be at most $\binom{n-1}{k-1}$. Manickam, Miklós, and Singhi conjectured that for $n \geq 4k$ this assignment gives the least possible number of nonnegative k-sums.

Conjecture 1 (Manickam, Miklós, Singhi, [2, 3])**.** *Suppose that $n \geq 4k$, and we have n real numbers $x_1, \ldots, x_n$ such that $x_1 + \cdots + x_n \geq 0$. Then, at least $\binom{n-1}{k-1}$ subsets $A \subset \{x_1, \ldots, x_n\}$ of order k satisfy $\sum_{a \in A} a \geq 0$*

Despite the apparent simplicity of the statement of Conjecture 1, it has been open for over two decades.

There have been several results establishing the conjecture when n is large compared to k. Manickam and Miklós [2] showed that the conjecture holds when $n \geq (k-1)(k^k + k^2) + k$ holds. Tyomkyn improved this bound to $n \geq k(4e \log k)^k \approx e^{ck \log \log k}$. Alon, Huang, and Sudakov [1] showed that the conjecture holds when $n \geq 33k^2$. Subsequently Frankl gave an alternative proof of the conjecture in a range of the form $n \geq 3k^3/2$.

We will talk about a proof of the conjecture in a range which is linear in k.

Theorem 1. *Suppose that $n \geq 10^{46}k$, and we have n real numbers $x_1, \ldots, x_n$ such that $x_1 + \cdots + x_n \geq 0$. At least $\binom{n-1}{k-1}$ subsets $A \subset \{x_1, \ldots, x_n\}$ of order k satisfy $\sum_{a \in A} a \geq 0$*

The method we use to prove Theorem 1 is inspired by an averaging argument which Katona used in his proof of the Erdős-Ko-Rado Theorem.

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On cliques in diameter graphs.

Andrey Kupavskii*, Alexandr Polyanskii†

March 14, 2014

Our talk is devoted to the study of the properties of cliques in diameter graphs.

Let us remind the definition of a diameter graph.

Definition. *A graph $G = (V, E)$ is a* **diameter graph** *in $\mathbb{R}^d$ (on S_r^d), if $V \subset \mathbb{R}^d$ (S_r^d) is a finite set of diameter 1 and edges of G are formed by vertices that are at unit distance apart.*

Note that we assume of the sphere being embedded in $\mathbb{R}^{d+1}$, and the unit distance included from the ambient space.

Diameter graphs arise naturally in the context of Borsuk's problem. In 1933 Borsuk [3] asked whether any set of diameter 1 in $\mathbb{R}^d$ can be partitioned into $(d+1)$ parts of strictly smaller diameter. The positive answer to this question is called Borsuk's conjecture. This was shown to be true in dimensions up to 3. In 1993 Kahn and Kalai [6] constructed a finite set of points in dimensions 1325 that does not admit a partition into 1326 parts of smaller diameter. The minimal dimension in which the counterexample is known is 64 (see [2], [5]).

We focus on one conjecture, posed by Morić and Pach [12].

Conjecture 1. *Any two d-cliques in a diameter graph in $\mathbb{R}^d$ must share at least $(d-2)$ vertices.*

This was proved for $d = 2$ by Hopf and Pannwitz in [7] and for $d = 3$ by V. Dol'nikov in [4]. As it is shown in [12], the following conjecture (Schur et al., [13]) reduces to Conjecture 1: in any diameter graph G in $\mathbb{R}^d$ the number of d-cliques is not greater than the number of vertices.

In [9] we proved Conjecture 1, which is the main topic of our talk. In the papers [1], [8], [10] we studied similar properties of cliques in diameter graphs in the space $\mathbb{R}^d$ and on the sphere S_r^d of radius $r > 1/\sqrt{2}$.

We note that some questions related to Conjecture 1 were studied in different terms by Maehara in [11]. In that paper he studies sphericity of complete bipartite graphs, where sphericity of a graph is the smallest dimension d such that the vertices of a graph can be represented by closed unit balls in $\mathbb{R}^d$ with two balls intersecting exactly if two corresponding vertices are adjacent. The result he obtained almost gives the following weaker version of Conjecture 1: any two d-cliques in a diameter graph in $\mathbb{R}^d$ must share at least one vertex.

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Zero-one law for random distance graphs with vertices in $\mathbb{Z}^n$

S.N.Popova [1]

Zero-one laws for random graphs have been considered for the first time by Glebskii Y. et al. in [1]. In this work the authors proved the zero-one law for Erdős–Rényi random graph $G(n,p)$. Later S. Shelah and J. Spencer expanded the class of functions $p(n)$, for which $G(n,p)$ follows the zero-one law (see [2]). M. Zhukovskii in [3] studied the zero-one law for random distance graphs with vertices being vectors from $\{0,1\}^n$ with equal numbers of zero and one coordinates. In [4] we considered a more general model — random distance graphs with vertices in $\{-1,0,1\}^n$, depending on a set of parameters.

Let us define the model of a random distance graph with vertices in $\mathbb{Z}^n$, generalizing the models from [3, 4]. Let $\mathcal{G}_n$ be the graph (V_n, E_n) with vertex set

$$V_n = \{\mathbf{v} = (v^1, \ldots, v^n) : |\{i \in \{1, \ldots, n\} : v^i = m\}| = a_m(n)\}$$

and edge set

$$E_n = \{\{\mathbf{u}, \mathbf{v}\} \in V_n \times V_n : (\mathbf{u}, \mathbf{v}) = c\},$$

where functions $a_m(n)$ satisfy the condition $\sum\limits_m a_m(n) = n$ and $(\mathbf{u}, \mathbf{v})$ is the Euclidean scalar product. The random distance graph with vertices in $\mathbb{Z}^n$ is the probabilistic space $G(\mathcal{G}_n, p) = (\Omega_{\mathcal{G}_n}, \mathcal{F}_{\mathcal{G}_n}, \mathsf{P}_{\mathcal{G}_n,p})$, where

$$\Omega_{\mathcal{G}_n} = \{G = (V, E) : V = V_n, E \subseteq E_n\},$$

$$\mathcal{F}_{\mathcal{G}_n} = 2^{\Omega_{\mathcal{G}_n}}, \quad \mathsf{P}_{\mathcal{G}_n,p}(G) = p^{|E|}(1-p)^{|E_n|-|E|}.$$

We prove the following results about the zero-one law for $G(\mathcal{G}_n, p)$.

Theorem 1. *Let $\sum\limits_m m a_m = kn$, $c(n) = k^2 n$, where $k \in \mathbb{Z}$ and $a_k(n) \to \infty$, $n \to \infty$. Then the random distance graph $\mathcal{G}(G_n, p)$ follows the zero-one law.*

Theorem 2. *Let $\sum\limits_m m a_m = \alpha n$, $c(n) = \alpha^2 n$, where $\alpha \in \mathbb{Q}$. Then there exists a subsequence $\mathcal{G}(G_{n_i}, p)$, following the zero-one law.*

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Upper bounds for some generalizations of the chromatic numbers.

Our talk will be devoted to some problems of Euclidian Ramsey theory. Let $C \subset \mathbb{R}^d$ be a finite point set. Denote by $\chi(C, n)$ the minimum number of colors needed to color $\mathbb{R}^d$ so that there is no monochromatic copy of C. C is called Ramsey, if $\chi(C, n) \to \infty$ as $n \to \infty$. The classical conjecture is that C is Ramsey iff $C \subset S^{d-1}$. This conjecture is proved for the vertex set of any simplex, products of Ramsey sets and some other sets. Moreover, Frankl and Rödl showed that $\chi(C, n)$ grows exponentially, provided C is the vertex set of a simplex. Recently, Raigorodsky, Zvonarev, Samirov, Harlamova gave accurate explicit bounds for the values of $\chi(C, n)$. A particular case of C is just a two point set. In this case $\chi(C, n)$ is the classical chromatic number $\chi(\mathbb{R}^n)$. An important generalization of $\chi(\mathbb{R}^n)$ is $\mathbb{R}^n_K$, where $\mathbb{R}^n_K$ is a normed space with a norm generated by a convex centrally-symmetric $K \subset \mathbb{R}^n$. Using our general approach based on Larman-Rogers' and Butler's technique, we prove that $\chi(\mathbb{R}^n_K) \leq (3 + o(1))^n$. This bound improves substantially previous results by Kang-Füredi and Kupavskii.

Graph Saturation Games

Ago-Erik Riet, University of Tartu

We study $\mathcal{F}$*-saturation games*, first introduced by Füredi, Reimer and Seress [1] in 1991, and named as such by West [2].
A graph G is H*-saturated* if H is not a subgraph of G, but adding any edge to G causes H to be a subgraph. We can ask what the minimum or maximum number of edges in an H-saturated graph on n vertices is - they are known as the *saturation number* and *Turán number (extremal number)*, respectively. Something that is naturally between those values is the *game saturation number* or *score*: two players, prolonger and shortener, start with an empty graph on n vertices and put down edges alternately, so that H is not a subgraph of the graph obtained during the game. Prolonger's strategy is to have as many edges as possible at the end and shortener has the opposite strategy. The game ends when the graph is H-saturated. The game saturation number or score is the length of the game or number of moves or number of edges at the end of the game.
We study the game saturation number for various graphs, digraphs or classes thereof. We show lower bounds on the length of path-avoiding games, and more precise results for short paths. We show sharp results for the tree avoiding game and the star avoiding game. We examine analogous games on directed graphs, and show tight results on the walk-avoiding game. We also examine an intermediate game played on undirected graphs, such that there exists an orientation avoiding a given family of directed graphs, and show bounds on the score. This is joint work with Jonathan Lee.

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On the Richter-Thomassen Conjecture about Pairwise Intersecting Curves

Natan Rubin*

Abstract

Let Γ be a family of n closed Jordan curves in the plane, where any two curves are either tangent or properly intersecting. We discuss a long standing conjecture of Richter and Thomassen which suggests, in its somewhat stronger form, that the overall number of intersection points among the curves of Γ must be strictly larger in asymptotic terms than the number of touching pairs of curves in Γ (as n tends to infinity).

We confirm the above conjecture in several important cases including x-monotone curves or, more generally, curves which can be decomposed into constantly many x-monotone pieces.

This is joint work in progress with János Pach and Gábor Tardos.

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Improved bounds for Pach's selection theorem

Jan Kynčl, Pavel Paták,
Zuzana Safernová, Martin Tancer

March 15, 2014

Abstract

We improve the estimates on the selection constant in the following geometric selection theorem by Pach: For every positive integer d there is a constant $c_d > 0$ such that whenever $X_1, \ldots, X_{d+1}$ are n-element subsets of $\mathbb{R}^d$, then we can find a point $\mathbf{p} \in \mathbb{R}^d$ and subsets $Y_i \subseteq X_i$ for every $i \in [d+1]$, each of size at least $c_d n$, such that $\mathbf{p}$ belongs to all *rainbow* d-simplices determined by $Y_1, \ldots, Y_{d+1}$, that is, simplices with one vertex in each Y_i.

We provide a lower bound $c_d > 2^{-2^{d^2+o(d)}}$, which is doubly exponentially decreasing in d (up to a polynomial in the exponent). For comparison, Pach's original approach yields a triply exponentially decreasing lower bound. We also show an exponentially decreasing upper bound $c_d \leq \kappa^d$ for a suitable constant $\kappa < 1$.

For the lower bound, we improve the 'separation' part of the argument by showing that in one of the key steps only $d+1$ separations are necessary, compared to 2^d separations in the original proof. In our construction for the upper bound, we use the fact that the minimum solid angle of every d-simplex is exponentially small. This fact was previously unknown and might be of independent interest.

Warning's Second Theorem with restricted variables

John R. Schmitt, Middlebury College

(joint work with Pete L. Clark (U. Georgia) and Aden Forrow (M.I.T.))

A well-known theorem of Chevalley states that a system of polynomials contained in an n-variable polynomial ring over a finite field of order q has a non-trivial zero whenever each polynomial has zero constant term and the sum of the degrees d is strictly less than n. In conjunction with this theorem is Warning's Theorem, which states, the number of shared zeros of such a polynomial system is divisible by the characteristic of the finite field. Less well-known is Warning's Second Theorem, which states, the number of shared zeros is at least q^{n-d}. We offer a new proof of this theorem using the polynomial method and a result of Alon and Füredi. We also provide a "restricted variables" generalization and show how this is a useful combinatorial tool.

Around Erdős–Lovász problem on colorings of non-uniform hypergraphs

Dmitry Shabanov

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Faculty of Mechanics and Mathematics
Department of Probability Theory

The talk deals with combinatorial problems concerning colorings of non-uniform hypergraphs. Let $H = (V, E)$ be a hypergraph with minimum edge-cardinality n. We show that if H is a simple hypergraph (i.e. every two distinct edges have at most one common vertex) and

$$\sum_{e\in E} r^{1-|e|} \leqslant c\sqrt{n},$$

for some absolute constant $c > 0$, then H is r-colorable. We also obtain a stronger result for triangle-free simple hypergraphs by proving that if H is a simple triangle-free hypergraph and

$$\sum_{e\in E} r^{1-|e|} \leqslant c \cdot n,$$

for some absolute constant $c > 0$, then H is r-colorable.

The work was partially supported by Russian Foundation of Fundamental Research (grant № 12-01-00683-a), by the program “Leading Scientific Schools” (grant no. NSh-2964.2014.1) and by the grant of the President of Russian Federation MK-692.2014.1

Subdivisions of a large clique in C_6-free graphs.

Maryam Sharifzadeh, UIUC

(Joint work with József Balogh and Hong Liu.)

Mader conjectured that every C_4-free graph has a subdivision of a clique of order linear in its average degree. We show that every C_6-free graph has such a subdivision of a large clique.

We also prove the dense case of Mader's conjecture in a stronger sense, i.e. for every c, there is a c' such that every C_4-free graph with average degree $cn^{1/2}$ has a subdivision of a clique K_ℓ with $\ell = \lfloor c'n^{1/2} \rfloor$ where every edge is subdivided exactly 3 times.

Algebraic techniques for combinatorial geometry: Recent developments

Micha Sharir, Tel Aviv University

In the past six years combinatorial geometry has experienced a major revolution, following the introduction of tools from algebraic geometry by Guth and Katz in 2008 and 2010. In the most exciting accomplishment of the new techniques, Guth and Katz have almost settled Erdős's problem on distinct distances in the plane, but many other significant developments have taken place since then. Many old problems have been solved, many improved bounds have been obtained, and the landscape of the field has considerably changed.

In this talk I will survey the recent developments. They include new bounds for other variants of the distinct distances problem, new bounds for incidences in various contexts, and re-examination of the theory of Elekes, Rónyai, and Szabó on polynomials vanishing on grids, and numerous applications thereof.

Some applications of Szemerédi-Trotter theorem to additive combinatorics

Ilya Shkredov, Steklov Instittue

We give a survey of recent applications of Szemerédi-Trotter theorem to problems concerning lower bounds of sumsets of convex sets, sizes of images of several convex maps and sum-product phenomena.

G-HAM SANDWICH THEOREMS: HARMONIC ANALYSIS AND MEASURE PARTITIONS

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Abstract. The Ham Sandwich Theorem – any n finite measures on $\mathbb{R}^n$ can be simultaneously bisected by single hyperplane – is the most classical result of equipartition theory, a topic central to geometric and topological combinatorics. We provide group-theoretic generalizations of this result, showing how finite measures can be "G-balanced" by unitary representations of a compact Lie group G. For abelian groups, such G-Ham Sandwich Theorems have an equivalent interpretation in terms of vanishing Fourier transforms. In the finite cases, these yield (equi-)partitions by families of complex regular q-fans of varying q, analogues of the famous Grünabum problem on equipartitions by families of hyperplanes (i.e., regular 2-fans). For the torus groups T^k, one has center transversal theorems in an L^2-sense for families of complex hyperplanes, similar in spirit to the center-point theorem of Rado.

Local chromatic number

Gábor Simonyi
Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences

The local chromatic number of graphs was introduced in an 1986 paper by Erdős, Füredi, Hajnal, Komjáth, Rödl. and Seress. It is the minimum number of colors that must appear in the most colorful closed neighborhood of a vertex in any proper coloring of the graph (with an arbitrary number of colors). It is (obviously) bounded from above by the chromatic number. Surprisingly, however, as proved in the above mentioned paper, for every $k \geq 3$ there exist graphs with local chromatic number 3 and chromatic number k. It was observed more recently, that the local chromatic number is bounded from below by the fractional chromatic number. This observation triggered the start of investigations of the local chromatic number for graphs with a large gap between their fractional chromatic number and (ordinary) chromatic number. There are not too many graph families known with this property, but those are usually "interesting" families of graphs. These include Kneser graphs and Schrijver graphs, generalised Mycielski graphs, shift graphs.

The talk tries to give a survey of results found in the last decade about the local chromatic number. It is based on joint papers with different subsets of the following co-authors: Bojan Mohar, János Körner, Concetta Pilotto, Gábor Tardos, Siniša Vrećica, Ambrus Zsbán.

Incidence problems in higher dimensions

József Solymosi, University of British Columbia

Geometric incidence problems have surprising applications in various fields of mathematics and theoretical computer science. The basic theorems bounding the maximum number of point-line and point-curve incidences are the Szemerédi-Trotter theorem (points and lines) and the Pach-Sharir theorem (curves and lines). There are many open questions for the planar case and even more for the higher dimensional variants. In this talk we will consider incidence problems in the real 3-space and in the complex plane.

Grid Ramsey problem and related questions

Benny Sudakov, ETH

(joint work with Conlon, Fox and Lee)

The Hales-Jewett theorem is one of the pillars of Ramsey theory, from which many other results follow. A celebrated result of Shelah says that Hales-Jewett numbers are primitive recursive. A key tool used in his proof, known as the cube lemma, has become famous in its own right. In its simplest form, it says that if we color the edges of the Cartesian product $K_n \times K_n$ in r colors then, for n sufficiently large, there is a rectangle with both pairs of opposite edges receiving the same color.

Hoping to improve Shelah's result, Graham, Rothschild and Spencer asked more than 20 years ago whether the cube lemma holds with n which is polynomial in r. We show that this is not possible by providing a superpolynomial lower bound in r. We also discuss a number of related questions, among them a solution of a problem of Erdős and Gyárfás on generalized Ramsey numbers.

Counting double-normal pairs

Konrad Swanepoel, London School of Economics

(joint work with János Pach)

Given a set of n points, there are various ways of declaring two points to be "far apart". Two well-known such notions are diameter pairs, where the distance between the points equals the diameter of the set (first considered by Erds), or antipodal pairs (introduced by Klee), where there exist parallel hyperplanes through the two points with the whole set contained in the closed slab bounded by the hyperplanes. Martini and Soltan (2005) introduced the notion of a double-normal pair, where we ask in addition to antipodality that the parallel hyperplanes are perpendicular to the line joining the two points. This notion interpolates between that of diameter pairs and antipodal pairs.

In this talk we discuss the problems of estimating the maximum number of diameters, antipodal pairs, or double-normal pairs in a set of n points in Euclidean space. The problems for diameters and antipodal pairs are well known, but nothing has previously been done for double-normal pairs.

Grid Ramsey problem and related questions

Endre Szemerédi, MTA Rényi Institute of Mathematics
(joint work with Asif Jamshed)

In 1974, Paul Seymour conjectured that any graph G of order n and minimum degree at least $\frac{k}{k+1}n$ contains the kth power of a Hamiltonian cycle. This conjecture was proved with the help of the Regularity Lemma for $n \geq n_0$ where n_0 is very large. Here we present another proof that avoids the use of the Regularity Lemma and thus the resulting n_0 is much smaller. The main ingredient is a new kind of connecting lemma.

Conflict-free coloring of graphs

Gábor Tardos, Rényi Institute, Budapest
(joint work with Roman Glebov and Tibor Szabó)

Conflict-free chromatic number of hypergraphs was introduced Even *et al.* and was motivated by a frecvency assignment problem. We study this parameter of the neighborhood hypergraphs of graphs from extremal and probabilistic points of view. We resolve a question of Pach and Tardos about the maximum conflict-free chromatic number the neighborhood hypergraph of an n-vertex graph can have. Our construction is randomized. In relation to this we study the evolution of the conflict-free chromatic number of the Erdős-Rényi random graph $G(n, p)$ and give the asymptotics for $p = \omega(1/n)$. We also show that for $p \geq 1/2$ the conflict-free chromatic number differs from the domination number by at most 3.

Intersecting hypergraphs

Norihide Tokushige, University of the Ryukyus

I will present some problems and results on extremal structures of hypergraphs satisfying some intersecting properties. Examples include extensions of Erdős-Ko-Rado theorem, Erdős's matching conjecture, and L-systems. I will also discuss tools such as Frankl's random walk method, and an extension of Hoffman's ratio bound.

Improved bounds on the partitioning of the Boolean lattice into chains of equal size

Istvan Tomon, Cambridge University

The Boolean lattice $2^{[n]}$ is the power set of $[n] = \{1, ..., n\}$ ordered by inclusion. We prove that if $n > 500c^2$ then $2^{[n]}$ can be partitioned into chains, with at most one exception each of length c. This improves a theorem of Lonc on the conjecture of Griggs. We also show that given a positive integer c and a poset P, whose Hesse diagram is connected then there exists $N(P, c)$ such that if $n > N(P, c)$ then the cartesian power P^n can be partitioned into chains, with at most one exception each of length c.

Making a C_6-free graph C_4-free and bipartite

Casey Tompkins
Joint work with Ervin Győri and Scott Kensell
Central European University

Let $e(G)$ denote the number of edges in a graph G, and let C_k denote a k-cycle. It is well-known that every graph has a bipartite subgraph with at least half as many edges. Győri showed that any bipartite, C_6-free graph contains a C_4-free subgraph containing at least half as many edges. Applying these two results in sequence we see that every C_6-free graph, G, has a bipartite C_4-free subgraph, H, with $e(H) \geq e(G)/4$. We show that the factor of $1/4$ can be improved to $3/8$:

Theorem 1. *Let G be a C_6-free graph, then G contains a subgraph with at least $3e(G)/8$ edges which is both C_4-free and bipartite.*

The proof uses probabilistic ideas combined with a charactarization of C_6-free graphs due to Füredi, Naor and Verstraëte.

Decomposing multiple coverings

Géza Tóth, MTA Rényi Institute of Mathematics

A planar set is cover-decomposable if a sufficiently thick covering of the plane by its translates can always be decomposed into two coverings. More than 30 years ago János Pach proposed the problem of determining cover-decomposable sets. He proved that centrally symmetric convex polygons are cover-decomposable. The problem is still not solved completely. We review the ideas of his proof, and many interesting improvements, generalizations, and related developments.

Matchings in balanced hypergraphs

Eberhard Triesch, RWTH Aachen (with R. Scheidweiler)

We investigate d-matchings and d-vertex covers in balanced hypergraphs $H = (V, E)$ where a weight function $d : E \to \mathbb{N}$ is given. The d-matching number $\Upsilon_d(H)$ is the maximum value of $\sum_{m \in M} d(m)$ where M is a matching in H. Some function $x : V \to \mathbb{N}$. is called a d-vertex cover if the inequality $\sum_{v \in e} x(v) \geq d(e)$ holds for every edge $e \in E$. The d-vertex cover number $\tau_d(H)$ is the minimal value of $\sum_{v \in V} x(v)$ where x is a d-vertex cover in H.

Berge and Las Vergnas (Annals of the New York Academy of Science, **175**, 1970, 32-40) proved what may be called Kőnig's Theorem for balanced hypergraphs, namely $\Upsilon_d(H) = \tau_d(H)$ for all weight functions $d : E \to \mathbb{N}$ Conforti, Cornuéjols Kapoor and Vušković (Combinatorica, **16**, 1996, 325-329) proved that the existence of a perfect matching is equivalent to the following analogue of Hall's condition: If some vertices are colored red and blue, and if there are more blue than red vertices in total, then there is an edge containing more blue than red vertices. This generalizes Hall's Theorem for bipartite graphs.

We prove a Min-Max Theorem which generalizes both results. In particular, we obtain a defect version of the generalized Hall Theorem. The proof is purely combinatorial.

Let $H = (V, E)$ denote a balanced hypergraph and assume that a second weight function $b : V \to \mathbb{N}$ is given. Define the weight of a partial hypergraph H' of H as

$$w(H') := \sum_{e \in E(H')} d(e) - \sum_{v \in V(H')} (\deg_{H'}(v) - 1) b(v).$$

Consider the optimization problem of maximizing $w(H')$ over all partial hypergraphs $H' \subseteq H$.

As a dual notion, let

$$X := X(H, d, b) := \{x | x \text{ is a } d - \text{vertex cover and } 0 \leq x(v) \leq b(v) \text{ for all } v \in V\}.$$

Main result:

Theorem 1. *Let $H = (V, E)$ be a balanced hypergraph and $d : E \to \mathbb{N}$ and $b : V \to \mathbb{N}$, such that for all $e \in E$: $\sum_{v \in e} b(v) \geq d(e)$. Then the following minimax-relation holds:*

$$\max_{H' \subseteq H} w(H') = \min_{x \in X} \sum_{v \in V} x(v)$$

Universality of graphs with few triangles and anti-triangles

Mykhaylo Tyomkyn

Call a graph sequence 3-*random-like* if it contains asymptotically the same number of triangles and empty 3-sets as the random graph $G_{n,1/2}$. This property is a natural relaxation of graph quasirandomness.

I will demonstrate that 3-random like graphs are 4-universal, meaning that each of them contains many induced copies of every 4-vertex graph. On the other hand, it is no longer true that 3-random like graphs are 5-universal. In fact, higher order universality can be disproved in a very strong sense.

Joint work with Dan Hefetz.

On the paintability number of $K_{n,n}$

Máté Vizer, MTA Rényi Institute of Mathematics
(joint work with Dániel Gerbner)

The notion of on-line choice number of a graph, which is called the paint number was introduced independently by Zhu and Schauz in 2009. It is natural to ask whether the difference between the choice and the paint number of a graph can be arbitrarily large. We do not answer this problem, however we show a new lower bound on the paint number of $K_{n,n}$, which is a good candidate to solve the question.

Searching d-defective sets with queries of size k

D. K. Vu*

Consider a set X of n elements. We wish to identify a particular subset Y containing at most d unknown elements. To this end, we perform a series of experiments with the following property: when testing a subset $A \subseteq X$, we receive a *positive* result if and only if A contains at least one of these d unknown elements. In practice, we often have the additional constraint that $|A| \leq k$, and we desire to minimize the total number of queries while yet determining Y exactly. This can be done adaptively, meaning that the answer of to a query influences which queries are made in the course of the search, or non-adaptively, where all questions are determined in advance. In the non-adaptive case, a successful family of such queries is often referred to as a *(d-)separating family.*

This question was first posed by A. Rényi in 1961. For the case of $d = 1$ G. O. H. Katona solved the adaptive case and provided upper and lower estimates for the non-adaptive case in 1966. In 2013, É. Hosszu, J. Tapolcai and G. Wiener simplified the proof remarkably. Using some of their ideas, we obtain similar results for general d. While the adaptive case is very similar, we also provide new (and to our knowledge the first non-trivial) upper and lower bounds in the non-adaptive case. We do so by examining the relationship between the girth of hypergraphs and separability.

In this talk the focus will be on the cases of $d = 2, 3$ for illustrative purposes.

This is joint work with F. S. Benevides, M. Delcourt, D. Gerbner, C. Palmer and B. Sinaimeri.

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Decomposition of multiple packings with subquadratic union complexity

Bartosz Walczak, EPFL

(joint work with János Pach)

Let k be a positive integer and $\mathcal{X}$ be a *k-fold packing* of simply connected compact sets in the plane, that is, a family such that every point belongs to at most k sets. Suppose that there is a function $f(n) = o(n^2)$ with the property that any n members of $\mathcal{X}$ surround at most $f(n)$ *holes*, which means that the complement of their union has at most $f(n)$ bounded connected components. We use tools from extremal graph theory and the topological Helly theorem to prove that $\mathcal{X}$ can be decomposed into at most p packings, where p is a constant depending only on k and f.

Let k be a positive integer and let X be a k-fold packing of simply connected compact sets in the plane, that is, a family such that every point belongs to at most k sets. Suppose that there is a function $f(n) = o(n^2)$ with the property that any n members of X surround at most $f(n)$ holes, which means that the complement of their union has at most f(n) bounded connected components. We use tools from extremal graph theory and the topological Helly theorem to prove that X can be decomposed into at most p packings, where p is a constant depending only on k and f.

Middle-Level Graphs

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In this note we examine the class of Middle-Level Graphs, $ML(k)$, also known as the Revolving Door Graphs. $ML(k)$ is defined as the subgraph of the $(2k+1)$-dimensional cube, $Q(2k+1)$, induced by the vertices with either exactly $k+1$ ones or exactly k ones. These graphs have been studied extensively in an attempt to settle the conjecture that they are Hamiltonian. They are known to be distance-transitive and therefore distance-regular. We will prove some results about their embedding in the cube and examine the middle level of the middle level. In particular we show that the middle level of ML(k) consists of disconnected copies of middle level graphs of lower dimension. Thus we show that the middle level of $ML(k)$ is the join of $\binom{k}{k/2}$ copies of $ML(k/2)$ when k is even and the join of $\binom{k+1}{(k+1)/2}$ copies of $ML((k-1)/2)$ when k is odd.

The Counting of Crossing-Free Geometric Graphs — Algorithms and Combinatorics

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Abstract

We are interested in the understanding of crossing-free geometric graphs–these are graphs with an embedding on a given planar point set where the edges are drawn as straight line segments without crossings. Often we are restricted to certain types of graphs, most prominently triangulations, but also spanning cycles, spanning trees, or (perfect) matchings (and crossing-free partitions), among others. A primary goal is to enumerate, to count, or to sample graphs of a certain type for a given point set–so these are algorithmic questions–, or to give estimates for the maximum and minimum number of such graphs on any set of n points–these are problems in extremal combinatorial geometry.

Among others, I will show some of the new ideas for providing extremal estimates, e.g. for the number of crossing-free spanning cycles: the support-refined estimate for cycles versus triangulations, the use of pseudo-simultaneously flippable edges in triangulations, and the employment of Kasteleyn's beautiful linear algebra method for counting perfect matchings in planar graphs–here, interestingly, in a weighted version. Moreover, Alvarez and Seidel's recent 2^n-algorithm for counting triangulations is discussed, with Wettstein's extensions to other types of graphs (e.g. crossing-free perfect matchings). This allows the first efficient enumeration algorithm (i.e. with polynomial delay) for crossing-free perfect matchings.

Keywords: computational geometry, geometric graphs, counting, sampling, enumeration

Zero-one k-laws for small k

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We study asymptotical behavior of the probabilities of first-order properties for Erdős–Rényi random graphs $G(n,p(n))$ with $p(n)=n^{-\alpha}$, $\alpha\in(0,1)$. The following zero-one law was proved in 1988 by S. Shelah and J.H. Spencer [1]: if α is irrational then for any first-order property L either the random graph satisfies the property L asymptotically almost surely or it doesn't satisfy (in such cases the random graph is said to *obey zero-one law*). When $\alpha\in(0,1)$ is rational the zero-one law for these graphs doesn't hold.

Let k be a positive integer. Denote by $\mathcal{L}_k$ the class of the first-order properties of graphs defined by formulae with quantifier depth bounded by the number k (the sentences are of a finite length). Let us say that the random graph obeys *zero-one k-law*, if for any first-order property $L\in\mathcal{L}_k$ either the random graph satisfies the property L almost surely or it doesn't satisfy. Since 2010 we prove several zero-one laws for rational α from $I_k=(0,\frac{1}{k-2}]\cup[1-\frac{1}{2^{k-1}},1)$. For some points from I_k we disprove the law. In particular, for $\alpha\in(0,\frac{1}{k-2})\cup(1-\frac{1}{2^k-2},1)$ zero-one k-law holds. If $\alpha\in\{\frac{1}{k-2},1-\frac{1}{2^k-2}\}$, then zero-one law does not hold (in such cases we call the number α k-critical).

From our results it follows that zero-one 3-law holds for any $\alpha\in(0,1)$. Therefore, there are no 3-critical points in $(0,1)$. Zero-one 4-law holds when $\alpha\in(0,1/2)\cup(13/14,1)$. Numbers 1/2 and 13/14 are 4-critical. Moreover, we know some rational 4-critical and not 4-critical numbers in $[7/8,13/14)$. Recently we obtain new results concerning zero-one 4-laws for $\alpha\in(1/2,7/8)$ and, thereby, narrow the gap.

References

[1] S. Shelah, J.H. Spencer, *Zero-one laws for sparse random graphs*, J. Amer. Math. Soc. **1**: 97–115, 1988.