

Grid Ramsey problem and related questions

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(joint work with Conlon, Fox and Lee)

The Hales-Jewett theorem is one of the pillars of Ramsey theory, from which many other results follow. A celebrated result of Shelah says that Hales-Jewett numbers are primitive recursive. A key tool used in his proof, known as the cube lemma, has become famous in its own right. In its simplest form, it says that if we color the edges of the Cartesian product $K_n \times K_n$ in r colors then, for n sufficiently large, there is a rectangle with both pairs of opposite edges receiving the same color.

Hoping to improve Shelah's result, Graham, Rothschild and Spencer asked more than 20 years ago whether the cube lemma holds with n which is polynomial in r . We show that this is not possible by providing a superpolynomial lower bound in r . We also discuss a number of related questions, among them a solution of a problem of Erdős and Gyárfás on generalized Ramsey numbers.