Local chromatic number

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The local chromatic number of graphs was introduced in an 1986 paper by Erdős, Füredi, Hajnal, Komjáth, Rödl. and Seress. It is the minimum number of colors that must appear in the most colorful closed neighborhood of a vertex in any proper coloring of the graph (with an arbitrary number of colors). It is (obviously) bounded from above by the chromatic number. Surprisingly, however, as proved in the above mentioned paper, for every $k \geq 3$ there exist graphs with local chromatic number 3 and chromatic number k. It was observed more recently, that the local chromatic number is bounded from below by the fractional chromatic number. This observation triggered the start of investigations of the local chromatic number for graphs with a large gap between their fractional chromatic number and (ordinary) chromatic number. There are not too many graph families known with this property, but those are usually "interesting" families of graphs. These include Kneser graphs and Schrijver graphs, generalised Mycielski graphs, shift graphs.

The talk tries to give a survey of results found in the last decade about the local chromatic number. It is based on joint papers with different subsets of the following co-authors: Bojan Mohar, János Körner, Concetta Pilotto, Gábor Tardos, Siniša Vrećica, Ambrus Zsbán.