

# G-HAM SANDWICH THEOREMS: HARMONIC ANALYSIS AND MEASURE PARTITIONS

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ABSTRACT. The Ham Sandwich Theorem – any  $n$  finite measures on  $\mathbb{R}^n$  can be simultaneously bisected by single hyperplane – is the most classical result of equipartition theory, a topic central to geometric and topological combinatorics. We provide group-theoretic generalizations of this result, showing how finite measures can be “ $G$ -balanced” by unitary representations of a compact Lie group  $G$ . For abelian groups, such  $G$ -Ham Sandwich Theorems have an equivalent interpretation in terms of vanishing Fourier transforms. In the finite cases, these yield (equi-)partitions by families of complex regular  $q$ -fans of varying  $q$ , analogues of the famous Grünbaum problem on equipartitions by families of hyperplanes (i.e., regular 2-fans). For the torus groups  $T^k$ , one has center transversal theorems in an  $L^2$ -sense for families of complex hyperplanes, similar in spirit to the center-point theorem of Rado.