Around Erdős–Lovász problem on colorings of non-uniform hypergraphs

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The talk deals with combinatorial problems concerning colorings of non-uniform hypergraphs. Let H = (V, E) be a hypergraph with minimum edge-cardinality n. We show that if H is a simple hypergraph (i.e. every two distinct edges have at most one common vertex) and

$$\sum_{e \in E} r^{1-|e|} \leqslant c\sqrt{n},$$

for some absolute constant c > 0, then H is r-colorable. We also obtain a stronger result for triangle-free simple hypergraphs by proving that if H is a simple triangle-free hypergraph and

$$\sum_{e \in E} r^{1-|e|} \leqslant c \cdot n,$$

for some absolute constant c > 0, then H is r-colorable.

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