Warning's Second Theorem with restricted variables

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A well-known theorem of Chevalley states that a system of polynomials contained in an n-variable polynomial ring over a finite field of order q has a non-trivial zero whenever each polynomial has zero constant term and the sum of the degrees d is strictly less than n. In conjunction with this theorem is Warning's Theorem, which states, the number of shared zeros of such a polynomial system is divisible by the characteristic of the finite field. Less well-known is Warning's Second Theorem, which states, the number of shared zeros is at least q^{n-d}. We offer a new proof of this theorem using the polynomial method and a result of Alon and Füredi. We also provide a "restricted variables" generalization and show how this is a useful combinatorial tool.