Graph Saturation Games

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We study *$\mathcal{F}$-saturation games*, first introduced by Füredi, Reimer and Seress [1] in 1991, and named as such by West [2].
A graph G is *H-saturated* if H is not a subgraph of G, but adding any edge to G causes H to be a subgraph. We can ask what the minimum or maximum number of edges in an H-saturated graph on n vertices is - they are known as the *saturation number* and *Turán number (extremal number)*, respectively. Something that is naturally between those values is the *game saturation number* or *score*: two players, prolonger and shortener, start with an empty graph on n vertices and put down edges alternately, so that H is not a subgraph of the graph obtained during the game. Prolonger's strategy is to have as many edges as possible at the end and shortener has the opposite strategy. The game ends when the graph is H-saturated. The game saturation number or score is the length of the game or number of moves or number of edges at the end of the game.
We study the game saturation number for various graphs, digraphs or classes thereof. We show lower bounds on the length of path-avoiding games, and more precise results for short paths. We show sharp results for the tree avoiding game and the star avoiding game. We examine analogous games on directed graphs, and show tight results on the walk-avoiding game. We also examine an intermediate game played on undirected graphs, such that there exists an orientation avoiding a given family of directed graphs, and show bounds on the score. This is joint work with Jonathan Lee.

References

[1] Füredi, Z., Reimer, D. and Seress, A., Triangle-Free Game and Extremal Graph Problems, *Congr. Numer.* **82** (1991), 123–128.

[2] West, D., The F-Saturation Game (2009) and Game Saturation Number (2011), *http://www.math.uiuc.edu/ west/regs/fsatgame.html* (last visited 11/25/2012).