Zero-one law for random distance graphs with vertices in $\mathbb{Z}^n$

S.N.Popova [1]

Zero-one laws for random graphs have been considered for the first time by Glebskii Y. et al. in [1]. In this work the authors proved the zero-one law for Erdős–Rényi random graph $G(n,p)$. Later S. Shelah and J. Spencer expanded the class of functions $p(n)$, for which $G(n,p)$ follows the zero-one law (see [2]). M. Zhukovskii in [3] studied the zero-one law for random distance graphs with vertices being vectors from $\{0,1\}^n$ with equal numbers of zero and one coordinates. In [4] we considered a more general model — random distance graphs with vertices in $\{-1,0,1\}^n$, depending on a set of parameters.

Let us define the model of a random distance graph with vertices in $\mathbb{Z}^n$, generalizing the models from [3, 4]. Let $\mathcal{G}_n$ be the graph (V_n, E_n) with vertex set

$$V_n = \{\mathbf{v} = (v^1, \ldots, v^n) : |\{i \in \{1, \ldots, n\} : v^i = m\}| = a_m(n)\}$$

and edge set

$$E_n = \{\{\mathbf{u}, \mathbf{v}\} \in V_n \times V_n : (\mathbf{u}, \mathbf{v}) = c\},$$

where functions $a_m(n)$ satisfy the condition $\sum_m a_m(n) = n$ and $(\mathbf{u}, \mathbf{v})$ is the Euclidean scalar product. The random distance graph with vertices in $\mathbb{Z}^n$ is the probabilistic space $G(\mathcal{G}_n, p) = (\Omega_{\mathcal{G}_n}, \mathcal{F}_{\mathcal{G}_n}, \mathsf{P}_{\mathcal{G}_n,p})$, where

$$\Omega_{\mathcal{G}_n} = \{G = (V, E) : V = V_n, E \subseteq E_n\},$$

$$\mathcal{F}_{\mathcal{G}_n} = 2^{\Omega_{\mathcal{G}_n}}, \quad \mathsf{P}_{\mathcal{G}_n,p}(G) = p^{|E|}(1-p)^{|E_n|-|E|}.$$

We prove the following results about the zero-one law for $G(\mathcal{G}_n, p)$.

Theorem 1. *Let $\sum_m m a_m = kn$, $c(n) = k^2 n$, where $k \in \mathbb{Z}$ and $a_k(n) \to \infty$, $n \to \infty$. Then the random distance graph $\mathcal{G}(G_n, p)$ follows the zero-one law.*

Theorem 2. *Let $\sum_m m a_m = \alpha n$, $c(n) = \alpha^2 n$, where $\alpha \in \mathbb{Q}$. Then there exists a subsequence $\mathcal{G}(G_{n_i}, p)$, following the zero-one law.*

---

[1] Moscow State University, Mechanics and Mathematics Faculty, Department of Mathematical Statistics and Random Processes

References

[1] Y.V. Glebskii, D.I. Kogan, M.I. Liagonkii, V.A. Talanov, *Range and degree of realizability of formulas the restricted predicate calculus*, Cybernetics 5: 142–154, 1969. (Russian original: Kibernetica 5, 17–27)

[2] S. Shelah, J.H. Spencer, *Zero–one laws for sparse random graphs*, J. Amer. Math. Soc. 1: 97–115, 1988.

[3] M.E. Zhukovskii, *On a sequence of random distance graphs subject to the zero-one law*, Problems of Information Transmission 47(3): 251-268, 2011. (Russian original: Problemy Peredachi Informatsii, 47(3): 39–58, 2011).

[4] S.N. Popova, *Zero-one law for random distance graphs with vertices in* $\{-1, 0, 1\}^n$, Problems of Information Transmission 50(1), 2014. (Russian original: Problemy Peredachi Informatsii, 50(1): 79–101, 2014).