1 Nonnegative k-sums in a set of numbers. Alexey Pokrovskiy

Suppose that we have a set of numbers $x_1, \dots, x_n$ which have nonnegative sum. How many subsets of k numbers from $\{x_1, \dots, x_n\}$ must have nonnegative sum?

By choosing $x_1 = n-1$ and $x_2 = \dots = x_n = -1$ we see that the answer to this question can be at most $\binom{n-1}{k-1}$. Manickam, Miklós, and Singhi conjectured that for $n \geq 4k$ this assignment gives the least possible number of nonnegative k-sums.

Conjecture 1 (Manickam, Miklós, Singhi, [2, 3])**.** *Suppose that $n \geq 4k$, and we have n real numbers $x_1, \dots, x_n$ such that $x_1 + \dots + x_n \geq 0$. Then, at least $\binom{n-1}{k-1}$ subsets $A \subset \{x_1, \dots, x_n\}$ of order k satisfy $\sum_{a \in A} a \geq 0$*

Despite the apparent simplicity of the statement of Conjecture 1, it has been open for over two decades.

There have been several results establishing the conjecture when n is large compared to k. Manickam and Miklós [2] showed that the conjecture holds when $n \geq (k-1)(k^k + k^2) + k$ holds. Tyomkyn improved this bound to $n \geq k(4e \log k)^k \approx e^{ck \log \log k}$. Alon, Huang, and Sudakov [1] showed that the conjecture holds when $n \geq 33k^2$. Subsequently Frankl gave an alternative proof of the conjecture in a range of the form $n \geq 3k^3/2$.

We will talk about a proof of the conjecture in a range which is linear in k.

Theorem 1. *Suppose that $n \geq 10^{46}k$, and we have n real numbers $x_1, \dots, x_n$ such that $x_1 + \dots + x_n \geq 0$. At least $\binom{n-1}{k-1}$ subsets $A \subset \{x_1, \dots, x_n\}$ of order k satisfy $\sum_{a \in A} a \geq 0$*

The method we use to prove Theorem 1 is inspired by an averaging argument which Katona used in his proof of the Erdős-Ko-Rado Theorem.

References

[1] N. Alon, H. Huang, and B. Sudakov., *Nonnegative k-sums, fractional covers, and probability of small deviations.* J. Combin Theory B, 102:784-796, (2012).

[2] N. Manickam and D. Miklós., *On the number of non-negative partial sums of a non-negative sum.* Colloq. Math. Soc. János Bolyai, 52:385-392, (1987).

[3] N. Manickam and N. Singhi., *First distribution invariants and EKR theorems.* J. Combin. Theory A, 48:91-103, (1988).