On the union of arithmetic progressions

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(joint work with Shoni Gilboa)

We show that for every $\epsilon > 0$ there is an absolute constant $c(\epsilon) > 0$ such that the following is true: The union of any n arithmetic progressions, each of length n, with pairwise distinct differences must consist of at least $c(\epsilon)n^{2-\epsilon}$ elements. We show also that this type of bound is essentially best possible, as we can find n arithmetic progressions, each of length n, with pairwise distinct differences such that the cardinality of their union is $o(n^2)$.
We develop some number theoretical tools that are of independent interest. In particular we give almost tight bounds on the following question: Given n distinct integers $a_1, ..., a_n$ at most how many pairs satisfy $a_j/a_i \in [n]$? More tight bounds on natural related problems will be presented.